Contents

Mathematicians Born in India

Apastamba

Born: bout 600 BC in India
Died: bout 600 BC in India

To write a biography of Apastamba is essentially impossible since nothing is known of him except that he was the author of a Sulbasutra which is certainly later than the Sulbasutra of Baudhayana. It would also be fair to say that Apastamba s Sulbasutra is the most interesting from a mathematical point of view. We do not know Apastamba s dates accurately enough to even guess at a life span for him, which is why we have given the same approximate birth year as death year.

Apastamba was neither a mathematician in the sense that we would understand it today, nor a scribe who simply copied manuscripts like Ahmes. He would certainly have been a man of very considerable learning but probably not interested in mathematics for its own sake, merely interested in using it for religious purposes. Undoubtedly he wrote the Sulbasutra to provide rules for religious rites and to improve and expand on the rules which had been given by his predecessors. Apastamba would have been a Vedic priest instructing the people in the ways of conducting the religious rites he describes.

The mathematics given in the Sulbasutras is there to enable the accurate construction of altars needed for sacrifices. It is clear from the writing that Apastamba, as well as being a priest and a teacher of religious practices, would have been a skilled craftsman. He must have been himself skilled in the practical use of the mathematics he described as a craftsman who himself constructed sacrificial altars of the highest quality.

The Sulbasutras are discussed in detail in the article Indian Sulbasutras. Below we give one or two details of Apastamba s Sulbasutra. This work is an expanded version of that of Baudhayana. Apastamba s work consisted of six chapters while the earlier work by Baudhayana contained only three.

The general linear equation was solved in the Apastamba s Sulbasutra. He also gives a remarkably accurate value for 2 namely

$$1 + 1/3 + 1/(3\times4) - 1/(3\times4\times34).$$

which gives an answer correct to five decimal places. A possible way that Apastamba might have reached this remarkable result is described in the article Indian Sulbasutras.

As well as the problem of squaring the circle, Apastamba considers the problem of dividing a segment into 7 equal parts. The article [3] looks in detail at a reconstruction of Apastamba s version of these two problems.

References for Apastamba

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Aryabhata the Elder

Born: 476 **in Kusumapura (now Patna), India**

Died: 550 **in India**

Aryabhata is also known as Aryabhata I to distinguish him from the later mathematician of the same name who lived about 400 years later. Al - Biruni has not helped in understanding Aryabhata s life, for he seemed to believe that there were two different mathematicians called Aryabhata living at the same time. He therefore created a confusion of two different Aryabhatas which was not clarified until 1926 when B Datta showed that al - Biruni s two Aryabhatas were one and the same person.

We know the year of Aryabhata s birth since he tells us that he was twenty - three years of age when he wrote Aryabhatiya which he finished in 499. We have given Kusumapura, thought to be close to Pataliputra (which was refounded as Patna in Bihar in 1541), as the place of Aryabhata s birth but this is far from certain, as is even the location of Kusumapura itself. As Parameswaran writes in [26]: -

> …*no final verdict can be given regarding the locations of Asmakajanapada and Kusumapura*.

We do know that Aryabhata wrote Aryabhatiya in Kusumapura at the time when Pataliputra was the capital of the Gupta empire and a major centre of learning, but there have been numerous other places proposed by historians as his birthplace. Some conjecture that he was born in south India, perhaps Kerala, Tamil Nadu or Andhra Pradesh, while others conjecture that he was born in the north - east of India, perhaps in Bengal. In [8] it is claimed that Aryabhata was born in the Asmaka region of the Vakataka dynasty in South India although the author accepted that he lived most of his life in Kusumapura in the Gupta empire of the north. However, giving Asmaka as Aryabhata s birthplace rests on a comment made by Nilakantha Somayaji in the late 15th century. It is now thought by most historians that Nilakantha confused Aryabhata with Bhaskara I who was a later commentator on the Aryabhatiya.

We should note that Kusumapura became one of the two major mathematical centres of India, the other being Ujjain. Both are in the north but Kusumapura (assuming it to be close to Pataliputra) is on the Ganges and is the more northerly. Pataliputra, being the capital of the Gupta empire at the time of Aryabhata, was the centre of a communications network which allowed learning from other parts of the world to reach it easily, and also allowed the mathematical and astronomical advances made by Aryabhata and his school to reach across India and also eventually into the Islamic world.

As to the texts written by Aryabhata only one has survived. However Jha claims in [21] that: -

> … *Aryabhata was an author of at least three astronomical texts and wrote some free stanzas as well.*

The surviving text is Aryabhata s masterpiece the Aryabhatiya which is a small astronomical treatise written in 118 verses giving a summary of Hindu mathematics up to that time. Its mathematical section contains 33 verses giving 66 mathematical rules without proof. The Aryabhatiya contains an introduction of 10 verses, followed by a section on mathematics with, as we just mentioned, 33 verses, then a section of 25 verses on the reckoning of time and planetary models, with the final section of 50 verses being on the sphere and eclipses.

Now there is a difficulty with this layout which is discussed in detail by van der Waerden in [35]. Van der Waerden suggests that in fact the 10 verse Introduction was written later than the other three sections. One reason for believing that the two parts were not intended as a whole is that the first section has a different meter to the remaining three sections. However, the problems do not stop there. We said that the first section had ten verses and indeed Aryabhata titles the section Set of ten giti stanzas. But it in fact contains eleven giti stanzas and two arya stanzas. Van der Waerden suggests that three verses have been added and he identifies a small number of verses in the remaining sections which he argues have also been added by a member of Aryabhata s school at Kusumapura.

The mathematical part of the Aryabhatiya covers arithmetic, algebra, plane trigonometry and spherical trigonometry. It also contains continued fractions, quadratic equations, sums of power series and a table of sines. Let us examine some of these in a little more detail.

First we look at the system for representing numbers which Aryabhata invented and used in the Aryabhatiya. It consists of giving numerical values to the 33 consonants of the Indian alphabet to represent 1, 2, 3, ⋯, 25, 30, 40, 50, 60, 70, 80, 90, 100. The higher numbers are denoted by these consonants followed by a vowel to obtain 100, 10000, ⋯ In fact the system allows numbers up to 10^{18} to be represented with an alphabetical notation. Ifrah in [3] argues that Aryabhata was also familiar with numeral symbols and the place - value system. He writes in [3]: -

⋯ it is extremely likely that Aryabhata knew the sign for zero and the numerals of the place value system. This supposition is based on the following two facts: first, the invention of his alphabetical counting system would have been impossible without zero or the place - value system; secondly, he carries out calculations on square and cubic roots which are impossible if the numbers in question are not written according to the place - value system and zero.

Next we look briefly at some algebra contained in the Aryabhatiya. This work is the first we are aware of which examines integer solutions to equations of the form by = ax + c and by = ax - c, where a, b, c are integers. The problem arose from studying the problem in astronomy of determining the periods of the planets. Aryabhata uses the kuttaka method to solve problems of this type. The word kuttaka means "to pulverise" and the method consisted of breaking the problem down into new problems where the coefficients became smaller and smaller with each step. The method here is essentially the use of the Euclidean algorithm to find the highest common factor of a and b but is also related to continued fractions.

Aryabhata gave an accurate approximation for π. He wrote in the Aryabhatiya the following: -

> *Add four to one hundred, multiply by eight and then add sixty-two thousand. the result is approximately the circumference of a circle of diameter twenty thousand. By this rule the relation of the circumference to diameter is given.*

This gives $\pi = 62832/20000 = 3.1416$ which is a surprisingly accurate value. In fact $\pi = 3.14159265$ correct to 8 places. If obtaining a value this accurate is surprising, it is perhaps even more surprising that Aryabhata does not use his accurate value for π but prefers to use $10 = 3.1622$ in practice. Aryabhata does not explain how he found this accurate value but, for example, Ahmad [5] considers this value as an approximation to half the perimeter of a regular polygon of 256 sides inscribed in the unit circle. However, in [9] Bruins shows that this result cannot be obtained from the doubling of the number of sides. Another interesting paper discussing this accurate value of π by Aryabhata is [22] where Jha writes: -

Aryabhata I s value of π is a very close approximation to the modern value and the most accurate among those of the ancients. There are reasons to believe that Aryabhata devised a particular method for finding this value. It is shown with sufficient grounds that Aryabhata himself used it, and several later Indian mathematicians and even the Arabs adopted it. The conjecture that Aryabhata s value of π is of Greek origin is critically examined and is found to be without foundation. Aryabhata discovered this value independently and also realised that π is an irrational number. He had the Indian background, no doubt, but excelled all his predecessors in evaluating π. Thus the credit of discovering this exact value of π may be ascribed to the celebrated mathematician, Aryabhata I.

We now look at the trigonometry contained in Aryabhata s treatise. He gave a table of sines calculating the approximate values at intervals of $90°/24 = 3°45'$. In order to do this he used a formula for $\sin(n+1)x - \sin nx$ in terms of $\sin nx$ and $\sin(n-1)x$. He also introduced the versine (versine = 1 - cosine) into trigonometry.

Other rules given by Aryabhata include that for summing the first n integers, the squares of these integers and also their cubes. Aryabhata gives formulas for the areas of a triangle and of a circle which are correct, but the formulas for the volumes of a sphere and of a pyramid are claimed to be wrong by most historians. For example Ganitanand in [15] describes as "mathematical lapses" the fact that Aryabhata gives the incorrect formula $V = Ah/2$ for the volume of a pyramid with height h and triangular base of area A. He also appears to give an incorrect expression for the volume of a sphere. However, as is often the case, nothing is as straightforward as it appears and Elfering (see for example [13]) argues that this is not an error but rather the result of an incorrect translation.

This relates to verses 6, 7, and 10 of the second section of the Aryabhatiya and in [13] Elfering produces a translation which yields the correct answer for both the volume of a pyramid and for a sphere. However, in his translation Elfering translates two technical terms in a different way to the meaning which they usually have. Without some supporting evidence that these technical terms have been used with these different meanings in other places it would still appear that Aryabhata did indeed give the incorrect formulas for these volumes.

Now we have looked at the mathematics contained in the Aryabhatiya but this is an astronomy text so we should say a little regarding the astronomy which it contains. Aryabhata gives a systematic treatment of the position of the planets in space. He gave the circumference of the earth as 4967 yojanas and its diameter as $1581\frac{1}{24}$ yojanas. Since 1 yojana = 5 miles this gives the circumference as 24835 miles, which is an excellent approximation to the currently accepted value of 24902 miles. He believed that the apparent rotation of the heavens was due to the axial rotation of the Earth. This is a quite remarkable view of the nature of the solar system which later commentators could not bring themselves to follow and most changed the text to save Aryabhata from what they thought were stupid errors!

Aryabhata gives the radius of the planetary orbits in terms of the radius of the Earth/Sun orbit as essentially their periods of rotation around the Sun. He believes that the Moon and planets shine by reflected sunlight, incredibly he believes that the orbits of the planets are ellipses. He correctly explains the causes of eclipses of the Sun and the Moon. The Indian belief up to that time was that eclipses were caused by a demon called Rahu. His value for the length of the year at 365 days 6 hours

12 minutes 30 seconds is an overestimate since the true value is less than 365 days 6 hours.

Bhaskara I who wrote a commentary on the Aryabhatiya about 100 years later wrote of Aryabhata: -

> *Aryabhata is the master who, after reaching the furthest shores and plumbing the inmost depths of the sea of ultimate knowledge of mathematics, kinematics and spherics, handed over the three sciences to the learned world.*

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Aryabhata II

Born: bout 920 in India

Died: bout 1000 in India

Essentially nothing is known of the life of Aryabhata II. Historians have argued about his date and have come up with many different theories. In [1] Pingree gives the date for his main publications as being between 950 and 1100. This is deduced from the usual arguments such as which authors Aryabhata II refers to and which refer to him. G R Kaye argued in 1910 that Aryabhata II lived before al - Biruni but Datta [2] in 1926 showed that these dates were too early.

The article [3] argues for a date of about 950 for Aryabhata II s main work, the Mahasiddhanta, but R Billiard has proposed a date for Aryabhata II in the sixteenth century. Most modern historians, however, consider the most likely dates for his main work as around 950 and we have given very approximate dates for his birth and death based on this hypothesis. See

[7] for a fairly recent discussion of this topic.

The most famous work by Aryabhata II is the Mahasiddhanta which consists of eighteen chapters. The treatise is written in Sanskrit verse and the first twelve chapters form a treatise on mathematical astronomy covering the usual topics that Indian mathematicians worked on during this period. The topics included in these twelve chapters are: the longitudes of the planets, eclipses of the sun and moon, the projection of eclipses, the lunar crescent, the rising and setting of the planets, conjunctions of the planets with each other and with the stars.

The remaining six chapters of the Mahasiddhanta form a separate part entitled on the sphere. It discusses topics such as geometry, geography and algebra with applications to the longitudes of the planets.

In Mahasiddhanta Aryabhata II gives in about twenty verses detailed rules to solve the indeterminate equation: $by = ax + c$. The rules apply in a number of different cases such as when c is positive, when c is negative, when the number of the quotients of the mutual divisions is even, when this number of quotients is odd, etc. Details of Aryabhata II s method are given in [6].

Aryabhata II also gave a method to calculate the cube root of a number, but his method was not new, being based on that given many years earlier by Aryabhata I, see for example [5].

Aryabhata II constructed a sine table correct up to five decimal places when measured in decimal parts of the radius, see [4]. Indian mathematicians were very interested in giving accurate sine tables since they were used to calculate the planetary positions as accurately as possible.

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Baudhayana

Born: bout 800 BC in India
Died: bout 800 BC in India

To write a biography of Baudhayana is essentially impossible since nothing is known of him except that he was the author of one of the earliest Sulbasutras. We do not know his dates accurately enough to even guess at a life span for him, which is why we have given the same approximate birth year as death year.

He was neither a mathematician in the sense that we would understand it today, nor a scribe who simply copied manuscripts like Ahmes. He would certainly have been a man of very considerable learning but probably not interested in mathematics for its own sake, merely interested in using it for religious purposes. Undoubtedly he wrote the Sulbasutra to provide rules for religious rites and it would appear an almost certainty that Baudhayana himself would be a Vedic priest.

The mathematics given in the Sulbasutras is there to enable the accurate construction of altars needed for sacrifices. It is clear from the writing that Baudhayana, as well as being a priest, must have been a skilled craftsman. He must have been himself skilled in the practical use of the mathematics he described as a craftsman who himself constructed sacrificial altars of the highest quality.

The Sulbasutras are discussed in detail in the article Indian Sulbasutras. Below we give one or two details of Baudhayana s Sulbasutra, which contained three chapters, which is the oldest which we possess and, it would be fair to say, one of the two most important.

The Sulbasutra of Baudhayana contains geometric solutions (but not algebraic ones) of a linear equation in a single unknown. Quadratic equations of the forms $ax^2 = c$ and $ax^2 + bx = c$ appear.

Several values of π occur in Baudhayana s Sulbasutra since when giving different constructions Baudhayana uses different approximations for constructing circular shapes. Constructions are given which are equivalent to taking π equal to 676/225 (where 676/225 = 3.004), 900/289 (where 900/289 = 3.114) and to 1156/361 (where 1156/361 = 3.202). None of these is particularly accurate but, in the context of constructing altars they would not lead to noticeable errors.

An interesting, and quite accurate, approximate value for 2 is given in Chapter 1 verse 61 of Baudhayana s Sulbasutra. The Sanskrit text gives in words what we would write in symbols as

$$2 = 1 + 1/3 + 1/(3\times 4) - 1/(3\times 4\times 34) = 577/408$$

which is, to nine places, 1.414215686. This gives 2 correct to five decimal places. This is surprising since, as we mentioned above, great mathematical accuracy did not seem necessary for the building work described. If the approximation was given as

$$2 = 1 + 1/3 + 1/(3\times 4)$$

then the error is of the order of 0.002 which is still more accurate than any of the values of π. Why then did Baudhayana feel that he had to go for a better approximation?

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Bhaskara I

Born: bout 600 in (possibly) Saurastra (modern Gujarat state), India

Died: bout 680 in (possibly) Asmaka, India

We have very little information about Bhaskara I s life except what can be deduced from his writings. Shukla deduces from the fact that Bhaskara I often refers to the Asmakatantra instead of the Aryabhatiya that he must have been working in a school of mathematicians in Asmaka which was probably in the Nizamabad District of Andhra Pradesh. If this is correct, and it does seem quite likely, then the school in Asmaka would have been a collection of scholars who were followers of Aryabhata I and of course this fits in well with the fact that Bhaskara I himself was certainly a follower of Aryabhata I.

There are other references to places in India in Bhaskara s writings. For example he mentions Valabhi (today Vala), the capital of the Maitraka dynasty in the 7th century, and Sivarajapura, which were both in Saurastra which today is the Gujarat state of India on the west coast of the continent. Also mentioned are Bharuch (or Broach) in southern Gujarat and Thanesar in the eastern Punjab which was ruled by Harsa for 41 years from 606. Harsa was the preeminent ruler in north India through the first half of Bhaskara I s life. A reasonable guess would be that Bhaskara was born in Saurastra and later moved to Asmaka.

Bhaskara I was an author of two treatises and commentaries to the work of Aryabhata I. His works are the Mahabhaskariya, the Laghubhaskariya and the Aryabhatiyabhasya. The Mahabhaskariya is an eight - chapter work on Indian mathematical astronomy and includes topics which were fairly standard for such works at this time. It discusses topics such as: the longitudes of the planets; conjunctions of the planets with each other and with bright stars; eclipses of the sun and the moon; risings and settings; and the lunar crescent.

Bhaskara I included in his treatise the Mahabhaskariya three verses which give an approximation to the trigonometric sine function by means of a rational fraction. These occur in Chapter 7 of the work. The formula which Bhaskara gives is amazingly accurate and use of the formula leads to a maximum error of less than one percent. The formula is

$$\sin x = 16x(\pi - x) / [5\pi^2 - 4x(\pi - x)]$$

and Bhaskara attributes the work as that of Aryabhata I. We have computed the values given by the formula and compared it with the correct value for sinx for x from 0 to $\pi/2$ in steps of $\pi/20$.

x = 0	formula = 0.00000	sinx = 0.00000	error = 0.00000
x = π/ 20	formula = 0.15800	sinx = 0.15643	error = 0.00157
x = π/ 10	formula = 0.31034	sinx = 0.30903	error = 0.00131
x = 3π/ 20	formula = 0.45434	sinx = 0.45399	error = 0.00035
x = π/ 5	formula = 0.58716	sinx = 0.58778	error = - 0.00062
x = π/ 4	formula = 0.70588	sinx = 0.70710	error = - 0.00122
x = π/ 10	formula = 0.80769	sinx = 0.80903	error = - 0.00134
x = 7π/ 20	formula = 0.88998	sinx = 0.89103	error = - 0.00105
x = 2π/ 5	formula = 0.95050	sinx = 0.95105	error = - 0.00055
x = 9π/ 20	formula = 0.98753	sinx = 0.98769	error = - 0.00016
x = π/ 2	formula = 1.00000	sinx = 1.00000	error = 0.00000

In 629 Bhaskara I wrote a commentary, the Aryabhatiyabhasya, on the Aryabhatiya by Aryabhata I. The Aryabhatiya contains 33 verses dealing with mathematics, the remainder of the work being concerned with mathematical astronomy. The

commentary by Bhaskara I is only on the 33 verses of mathematics. He considers problems of indeterminate equations of the first degree and trigonometric formulas. In the course of discussions of the Aryabhatiya, Bhaskara I expressed his idea on how one particular rectangle can be treated as a cyclic quadrilateral. He was the first to open discussion on quadrilaterals with all the four sides unequal and none of the opposite sides parallel.

One of the approximations used for π for many centuries was 10. Bhaskara I criticised this approximation. He regretted that an exact measure of the circumference of a circle in terms of diameter was not available and he clearly believed that π was not rational.

In [11], [12], [13] and [14] Shukla discusses some features of Bhaskara s mathematics such as: numbers and symbolism, the classification of mathematics, the names and solution methods of equations of the first degree, quadratic equations, cubic equations and equations with more than one unknown, symbolic algebra, unusual and special terms in Bhaskara s work, weights and measures, the Euclidean algorithm method of solving linear indeterminate equations, examples given by Bhaskara I illustrating Aryabhata I s rules, certain tables for solving an equation occurring in astronomy, and reference made by Bhaskara I to the works of earlier Indian mathematicians.

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Bhaskara

Born: 1114 in Vijayapura, India
Died: 1185 in Ujjain, India

Bhaskara is also known as Bhaskara II or as Bhaskaracharya, this latter name meaning "Bhaskara the Teacher". Since he is known in India as Bhaskaracharya we will refer to him throughout this article by that name. Bhaskaracharya s father was a Brahman named Mahesvara. Mahesvara himself was famed as an astrologer. This happened frequently in Indian society with generations of a family being excellent mathematicians and often acting as teachers to other family members.

Bhaskaracharya became head of the astronomical observatory at Ujjain, the leading mathematical centre in India at that time. Outstanding mathematicians such as Varahamihira and Brahmagupta had worked there and built up a strong school of mathematical astronomy.

In many ways Bhaskaracharya represents the peak of mathematical knowledge in the 12th century. He reached an understanding of the number systems and solving equations which was not to be achieved in Europe for several centuries.

Six works by Bhaskaracharya are known but a seventh work, which is claimed to be by him, is thought by many historians to be a late forgery. The six works are: Lilavati (The Beautiful) which is on mathematics; Bijaganita (Seed Counting or Root Extraction) which is on algebra; the Siddhantasiromani which is in two parts, the first on mathematical astronomy with the second part on the sphere; the Vasanabhasya of Mitaksara which is Bhaskaracharya s own commentary on the Siddhantasiromani; the Karanakutuhala (Calculation of Astronomical Wonders) or Brahmatulya which is a simplified version of the Siddhantasiromani; and the Vivarana which is a commentary on the Shishyadhividdhidatantra of Lalla. It is the first three of these works which are the most interesting, certainly from the point of view of mathematics, and we will concentrate on the contents of these.

Given that he was building on the knowledge and understanding of Brahmagupta it is not surprising that Bhaskaracharya understood about zero and negative numbers. However his understanding went further even than that of Brahmagupta. To give some examples before we examine his work in a little more detail we note that he knew that $x^2 = 9$ had two solutions.

He also gave the formula

$$\sqrt{a \pm \sqrt{b}} = \sqrt{\frac{a + \sqrt{a^2 - b}}{2}} \pm \sqrt{\frac{a - \sqrt{a^2 - b}}{2}}$$

Bhaskaracharya studied Pell s equation $px^2 + 1 = y^2$ for p = 8, 11, 32, 61 and 67. When p = 61 he found the solutions x = 226153980, y = 1776319049. When p = 67 he found the solutions x = 5967, y = 48842. He studied many Diophantine problems.

Let us first examine the Lilavati. First it is worth repeating the story told by Fyzi who translated this work into Persian in 1587. We give the story as given by Joseph in [5]: -

> *Lilavati was the name of Bhaskaracharya s daughter. From casting her horoscope, he discovered that the auspicious time for her wedding would be a particular hour on a certain day. He placed a cup with a small hole at the bottom of the vessel filled with water, arranged so that the cup would sink at the beginning of the propitious hour. When everything was ready and the cup was placed in the vessel, Lilavati suddenly out of curiosity bent over the vessel and a pearl from her dress fell into the cup and blocked the hole in it. The lucky hour passed without the cup sinking. Bhaskaracharya be-*

lieved that the way to console his dejected daughter, who now would never get married, was to write her a manual of mathematics!

This is a charming story but it is hard to see that there is any evidence for it being true. It is not even certain that Lilavati was Bhaskaracharya s daughter. There is also a theory that Lilavati was Bhaskaracharya s wife. The topics covered in the thirteen chapters of the book are: definitions; arithmetical terms; interest; arithmetical and geometrical progressions; plane geometry; solid geometry; the shadow of the gnomon; the kuttaka; combinations.

In dealing with numbers Bhaskaracharya, like Brahmagupta before him, handled efficiently arithmetic involving negative numbers. He is sound in addition, subtraction and multiplication involving zero but realised that there were problems with Brahmagupta s ideas of dividing by zero. Madhukar Mallayya in [14] argues that the zero used by Bhaskaracharya in his rule $(a.0)/0 = a$, given in Lilavati, is equivalent to the modern concept of a non - zero "infinitesimal". Although this claim is not without foundation, perhaps it is seeing ideas beyond what Bhaskaracharya intended.

Bhaskaracharya gave two methods of multiplication in his Lilavati. We follow Ifrah who explains these two methods due to Bhaskaracharya in [4]. To multiply 325 by 243 Bhaskaracharya writes the numbers thus:

243 243 243
3 2 5

Now working with the rightmost of the three sums he computed 5 times 3 then 5 times 2 missing out the 5 times 4 which he did last and wrote beneath the others one place to the left. Note that this avoids making the "carry" in ones head.

243 243 243
3 2 5

1015
20

Now add the 1015 and 20 so positioned and write the answer under the second line below the sum next to the left.

243 243 243
3 2 5

1015
20

1215

Work out the middle sum as the right - hand one, again avoiding the " carry ", and add them writing the answer below the 1215 but displaced one place to the left .

243 243 243

3 2 5

---

4 6 1015

8 20

1215

486

Finally work out the left most sum in the same way and again place the resulting addition one place to the left under the 486 .

243 243 243

3 2 5

---

6 9 4 6 1015

12 8 20

1215

486

729

Finally add the three numbers below the second line to obtain the answer 78975 .

243 243 243

3 2 5

---

6 9 4 6 1015

12 8 20

1215

486

729

78975

Despite avoiding the" carry" in the first stages, of course one is still faced with the" carry" in this final addition .

The second of Bhaskaracharya s methods proceeds as follows:

325

243

Multiply the bottom number by the top number starting with the left - most digit and proceeding towards the right. Displace each row one place to start one place further right than the previous line. First step

325

243

729

Second step

325

243

729

486

Third step, then add

325

243

729

486

1215

78975

Bhaskaracharya, like many of the Indian mathematicians, considered squaring of numbers as special cases of multiplication which deserved special methods. He gave four such methods of squaring in Lilavati.

Here is an example of explanation of inverse proportion taken from Chapter 3 of the Lilavati. Bhaskaracharya writes: -

In the inverse method, the operation is reversed. That is the fruit to be multiplied by the augment and divided by the demand. When fruit increases or decreases, as the demand is augmented or diminished, the direct rule is used. Else the inverse.

Rule of three inverse: If the fruit diminish as the requisition increases, or augment as that decreases, they, who are skilled in accounts, consider the rule of three to be inverted. When there is a diminution of fruit, if there be increase of requisition, and increase of fruit if there be diminution of requisition, then the inverse rule of three is employed.

As well as the rule of three, Bhaskaracharya discusses examples to illustrate rules of compound proportions, such as the rule of five (Pancarasika), the rule of seven (Saptarasika), the rule of nine (Navarasika), etc. Bhaskaracharya s examples of using these rules are discussed in [15].

An example from Chapter 5 on arithmetical and geometrical progressions is the following: -

Example: On an expedition to seize his enemy s elephants, a king marched two yojanas the first day. Say, intelligent calculator, with what increasing rate of daily march did he proceed, since he reached his foe s city, a distance of eighty yojanas, in a week?

Bhaskaracharya shows that each day he must travel 22/7 yojanas further than the previous day to reach his foe s city in 7 days.

An example from Chapter 12 on the kuttaka method of solving indeterminate equations is the following: -

Example: Say quickly, mathematician, what is that multiplier, by which two hundred and twenty-one being multiplied, and sixty-five added to the product, the sum divided by a hundred and ninety-five becomes exhausted.

Bhaskaracharya is finding integer solution to $195x = 221y + 65$. He obtains the solutions $(x, y) = (6, 5)$ or $(23, 20)$ or $(40, 35)$ and so on.

In the final chapter on combinations Bhaskaracharya considers the following problem. Let an n-digit number be represented in the usual decimal form as

(*) $d_1 d_2 \cdots d_n$

where each digit satisfies $1 \le d_j \le 9$, $j = 1, 2, \cdots, n$. Then Bhaskaracharya s problem is to find the total number of numbers of the form (*) that satisfy

$d_1 + d_2 + \cdots + d_n = S$.

In his conclusion to Lilavati Bhaskaracharya writes: -

Joy and happiness is indeed ever increasing in this world for those who have Lilavati clasped to their throats, decorated as the members are with neat reduction of fractions, multiplication and involution, pure and perfect as are the solutions, and tasteful as is the speech which is exemplified.

The Bijaganita is a work in twelve chapters. The topics are: positive and negative numbers; zero; the unknown; surds; the kuttaka; indeterminate quadratic equations; simple equations; quadratic equations; equations with more than one unknown; quadratic equations with more than one unknown; operations with products of several unknowns; and the author and his work.

Having explained how to do arithmetic with negative numbers, Bhaskaracharya gives problems to test the abilities of the reader on calculating with negative and affirmative quantities: -

Example: Tell quickly the result of the numbers *three and four, negative or affirmative, taken together; that is, affirmative and negative, or both negative or both affirmative, as separate instances; if thou know the addition of affirmative and negative quantities.*

Negative numbers are denoted by placing a dot above them: -

The characters, denoting the quantities known and unknown, should be first written to indicate them generally; and those, which become negative should be then marked with a dot over them.

Example: Subtracting two from three, affirmative from affirmative, and negative from negative, or the contrary, tell me quickly the result ...

In Bijaganita Bhaskaracharya attempted to improve on Brahmagupta s attempt to divide by zero (and his own description in Lilavati) when he wrote: -

A quantity divided by zero becomes a fraction the denominator of which is zero. This fraction is termed an infinite quantity. In this quantity consisting of that which has zero for its divisor, there is no alteration, though many may be inserted or ex-

tracted; as no change takes place in the infinite and immutable God when worlds are created or destroyed, though numerous orders of beings are absorbed or put forth.

So Bhaskaracharya tried to solve the problem by writing $n/0 = \infty$. At first sight we might be tempted to believe that Bhaskaracharya has it correct, but of course he does not. If this were true then 0 times must be equal to every number n, so all numbers are equal. The Indian mathematicians could not bring themselves to the point of admitting that one could not divide by zero.

Equations leading to more than one solution are given by Bhaskaracharya: -

Example: Inside a forest, a number of apes equal to the square of one - eighth of the total apes in the pack are playing noisy games. The remaining twelve apes, who are of a more serious disposition, are on a nearby hill and irritated by the shrieks coming from the forest. What is the total number of apes in the pack?

The problem leads to a quadratic equation and Bhaskaracharya says that the two solutions, namely 16 and 48, are equally admissible.

The kuttaka method to solve indeterminate equations is applied to equations with three unknowns. The problem is to find integer solutions to an equation of the form $ax + by + cz = d$. An example he gives is: -

Example: The horses belonging to four men are 5, 3, 6 *and* 8. *The camels belonging to the same men are* 2, 7, 4 *and* 1. *The mules belonging to them are* 8, 2, 1 *and* 3 *and the oxen are* 7, 1, 2 *and* 1. *All four men have equal fortunes. Tell me quickly the price of each horse, camel, mule and ox.*

Of course such problems do not have a unique solution as Bhaskaracharya is fully aware. He finds one solution, which is the minimum, namely horses 85, camels 76, mules 31 and oxen 4.

Bhaskaracharya s conclusion to the Bijaganita is fascinating for the insight it gives us into the mind of this great mathematician: -

A morsel of tuition conveys knowledge to a comprehensive mind; and having reached it, expands of its own impulse, as oil poured upon water, as a secret entrusted to the vile, as alms bestowed upon the worthy, however little, so does knowledge infused into a wise mind spread by intrinsic force.

It is apparent to men of clear understanding, that the rule of three terms constitutes arithmetic and sagacity constitutes algebra. Accordingly I have said … The rule of three terms is arithmetic; spotless understanding is algebra. What is there unknown to the intelligent? Therefore for the dull alone it is set forth.

The Siddhantasiromani is a mathematical astronomy text similar in layout to many other Indian astronomy texts of this and earlier periods. The twelve chapters of the first part cover topics such as: mean longitudes of the planets; true longitudes of the planets; the three problems of diurnal rotation; syzygies; lunar eclipses; solar eclipses; latitudes of the planets; risings and settings; the moon s crescent; conjunctions of the planets with each other; conjunctions of the planets with the fixed stars; and the patas of the sun and moon.

The second part contains thirteen chapters on the sphere. It covers topics such as: praise of study of the sphere; nature of the sphere; cosmography and geography; planetary mean motion; eccentric epicyclic model of the planets; the armillary sphere; spherical trigonometry; ellipse calculations; first visibilities of the planets; calculating the lunar crescent; astronomical instruments; the seasons; and problems of astronomical calculations.

There are interesting results on trigonometry in this work. In particular Bhaskaracharya seems more interested in trigonometry for its own sake than his predecessors who saw it only as a tool for calculation. Among the many interesting results given by Bhaskaracharya are:

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

and

$$\sin(a-b) = \sin a \cos b - \cos a \sin b.$$

Bhaskaracharya rightly achieved an outstanding reputation for his remarkable contribution. In 1207 an educational institution was set up to study Bhaskaracharya s works. A medieval inscription in an Indian temple reads: -

Triumphant is the illustrious Bhaskaracharya whose feats are revered by both the wise and the learned. A poet endowed with fame and religious merit, he is like the crest on a peacock.

It is from this quotation that the title of Joseph s book [5] comes.

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Satyendranath Bose

Born: 1 Jan 1894 **in Calcutta, India**

Died: 4 **Feb** 1974 **in Calcutta, India**

Satyendranath Bose s mother, Amodini Devi, had received little formal education but she skilfully brought up her large family of seven children. Bose s father was Surendranath Bose who worked for a while as an accountant before joining the East Indian Railways. He later set up his own chemical and pharmaceutical company. Satyendranath was the eldest of Amodini and Surendranath s seven children, having six younger sisters.

Satyendranath began his education at an elementary school in Calcutta before entering the Hindu School in 1907. It was here that his interest in mathematics and science began, and as is so often the case, it was due to an outstanding mathematics teacher coupled with encouragement from the headmaster.

He began his studies at Presidency College, Calcutta, in 1909 where he had a brilliant academic record. He was awarded a B.Sc. in 1913 and an M.Sc. in 1915 proving himself to be by far the best student of mathematics. In the year he was awarded his Master s degree, Bose married Ushabala Ghosh. They had five children, three daughters and two sons.

Had Indians been allowed to take administrative posts in the government service, Bose would almost certainly have followed that route. As it was, he continued to study physics and mathematics and was appointed to the newly opened University College of Science in Calcutta in 1917. This university was a research institution for postgraduate studies and here Bose was able to study recent European texts on quantum theory and relativity which, before the opening of the new institution, had not been readily available in India. Gibbs book on statistical mechanics stimulated Bose s interest in this topic. He also studied Einstein s papers on relativity and obtained Einstein s permission to translate them for publication in India.

Bose was appointed as a Reader in physics at the University of Dacca in 1921 and taught there until 1945, being a professor and head of the physics department from 1927 . In 1945 he returned to Calcutta University when he was appointed as Guprasad Sing Professor of Physics, a position he held until he retired in 1956 when he was made Professor Emeritus .

He did important work in quantum theory, in particular on Planck s black body radiation law . Bose sent his paper Planck s Law and the Hypothesis of Light Quanta (1924) to Einstein . He wrote a covering letter saying: -

> *Respected Sir, I have ventured to send you the accompanying article for your perusal and opinion . You will see that I have tried to deduce the coeffi-cient . in Planck s law independent of classical electrodynamics .*

This paper was only four pages long but it was highly significant . The derivation of Planck s formula had not been to Planck s satisfaction, and Einstein too was unhappy with it . Now Bose was able to derive the formula for radiation from Boltzmann s statistics . The paper, and his method of deriving Planck s radiation formula, was enthusiastically endorsed by Einstein who saw at once that Bose had removed a major objection against light quanta . The paper was translated into German by Einstein and submitted with a strong recommendation to the Zeitschrift für Physik . Einstein extended Bose s treatment to material particles whose number is conserved and published several papers on this extension.

An important consequence of Einstein s response to Bose s article was that his application to the University of Dacca for two years research leave beginning in 1924 was approved. He now had the chance of meeting European scientists and travelled first to Paris where he met Langevin and de Broglie. In October 1925 Bose travelled from Paris to Berlin where he met Einstein. Much progress had been made by Einstein following his receipt of Bose s paper for he was able to see how the ideas could be taken forward. While he was in Berlin Bose attended a course on quantum theory given by Born.

Bose published on statistical mechanics leading to the Einstein - Bose statistics. Dirac coined the term boson for particles obeying these statistics. Through these terms his name is rightly known and remembered, for indeed his contributions are remarkable, especially given the fact that he made his important discoveries working in isolation from the mainstream developments in Europe.

It was not only for his research contributions that Bose is important, however, for his efforts to improve education in India led to a much greater use of technology. He gave leadership in many ways: as president of the physics section of the Indian

Science Congress in 1939, as general president of the Indian Science Congress in Delhi in 1944, and as president of the National Institute of Science of India in 1949. His greatest honour was election to the Royal Society of London in 1958.

After Bose retired from Calcutta University in 1956 he was appointed as vice - chancellor of Viswa - Bharati University, Santiniketan. Two years later he was honoured with the post of national professor.

P T Landsberg writes [3]: -

> *The high regard with which [Bose] was held in India can hardly be appreciated in the West, where respect for old age is much less developed than it is in India. Bose s shock of white hair and friendly personality was probably last in evidence ant a public function in January of this year, when an international symposium on statistical physics was held in Calcutta. Special references were made to his famous paper, and Bose himself also addressed the meeting, asking his colleagues to keep a fresh "that wonderful spark" which gave fulfilment to scientific work.*

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Brahmadeva

Born: bout 1060 in (possibly) Mathura, India
Died: bout 1130 in India

Brahmadeva was the son of Candrabudha. The family came from the Mathura district of Uttar Pradesh in northern India. He was born into the Brahman caste which meant he was from the highest ranking caste of Hindu priests. The Brahman caste had a strong tradition of education so Brahmadeva would have received one of the best educations of men of his time.

We have only one work by Brahmadeva and this is Karanaprakasa which is a commentary on the Aryabhatiya by Aryabhata I. Brahmadeva s work is in nine chapters and it follows the contents of the original Aryabhatiya. Topics covered include the longitudes of the planets, problems relating to the daily rotation of the heavens, eclipses of the sun and the moon, risings and settings, the lunar crescent, and conjunctions of the planets.

The work contains some contributions to trigonometry, motivated by its application to mathematical astronomy. It is this aspect of the work which is mentioned by Gupta in [2].

Different commentaries on the Aryabhatiya achieved popularity in different parts of India. Brahmadeva s commentary seems to have been particularly popular in Madras, Mysore and Maharastra. The more important commentaries on the Aryabhatiya became the basis for further commentaries and indeed this is what happened to the Karanaprakasa. Commentaries on Brahmadeva s work continued to appear up to the seventeenth century.

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Brahmagupta

Born: 598 **in (possibly) Ujjain, India**
Died: 670 **in India**

Brahmagupta, whose father was Jisnugupta, wrote important works on mathematics and astronomy. In particular he wrote Brahmasphutasiddhanta (The Opening of the Universe), in 628. The work was written in 25 chapters and Brahmagupta told us in the text that he wrote it at Bhillamala which today is the city of Bhinmal. This was the capital of the lands ruled by the Gurjara dynasty.

Brahmagupta became the head of the astronomical observatory at Ujjain which was the foremost mathematical centre of ancient India at this time. Outstanding mathematicians such as Varahamihira had worked there and built up a strong school of mathematical astronomy.

In addition to the Brahmasphutasiddhanta Brahmagupta wrote a second work on mathematics and astronomy which is the Khandakhadyaka written in 665 when he was 67 years old.

We look below at some of the remarkable ideas which Brahmagupta s two treatises contain . First let us give an overview of their contents .

The Brahmasphutasiddhanta contains twenty - five chapters but the first ten of these chapters seem to form what many historians believe was a first version of Brahmagupta s work and some manuscripts exist which contain only these chapters . These ten chapters are arranged in topics which are typical of Indian mathematical astronomy texts of the period . The topics covered are: mean longitudes of the planets; true longitudes of the planets; the three problems of diurnal rotation; lunar eclipses; solar eclipses; risings and settings; the moon s crescent; the moon s shadow; conjunctions of the planets with each other; and conjunctions of the planets with the fixed stars .

The remaining fifteen chapters seem to form a second work which is major addendum to the original treatise . The chapters are: examination of previous treatises on astronomy; on mathematics; additions to chapter 1; additions to chapter 2; additions to chapter 3; additions to chapter 4 and 5; additions to chapter 7; on algebra; on the gnomon; on meters; on the sphere; on instruments; summary of contents; versified tables .

Brahmagupta s understanding of the number systems went far beyond that of others of the period. In the Brahmasphuta-siddhanta he defined zero as the result of subtracting a number from itself. He gave some properties as follows: -

When zero is added to a number or subtracted from a number, the number remains unchanged; and a number multiplied by zero becomes zero.

He also gives arithmetical rules in terms of fortunes (positive numbers) and debts (negative numbers): -

A debt minus zero is a debt.

A fortune minus zero is a fortune.

Zero minus zero is a zero.

A debt subtracted from zero is a fortune.

A fortune subtracted from zero is a debt.

The product of zero multiplied by a debt or fortune is zero.

The product of zero multiplied by zero is zero.

The product or quotient of two fortunes is one fortune.

The product or quotient of two debts is one fortune.

The product or quotient of a debt and a fortune is a debt.

The product or quotient of a fortune and a debt *is a debt*.

Brahmagupta then tried to extend arithmetic to include division by zero: -

Positive or negative numbers when divided by zero is a fraction the zero as denominator.

Zero divided by negative or positive numbers is either zero or is expressed as a fraction with zero as numerator and the finite quantity as denominator.

Zero divided by zero is zero.

Really Brahmagupta is saying very little when he suggests that n divided by zero is $n/0$. He is certainly wrong when he then claims that zero divided by zero is zero. However it is a brilliant attempt to extend arithmetic to negative numbers and zero.

We can also describe his methods of multiplication which use the place - value system to its full advantage in almost the same way as it is used today. We give three examples of the methods he presents in the Brahmasphuta siddhanta and in doing so we follow Ifrah in [4]. The first method we describe is called" gomutrika "by Brahmagupta. Ifrah translates" gomutrika "to" like the trajectory of a cow s urine ". Consider the product of 235 multiplied by 264. We begin by setting out the sum as follows:

2 235

6 235

4 235

Now multiply the 235 of the top row by the 2 in the top position of the left hand column. Begin by $2 \times 5 = 10$, putting 0 below the 5 of the top row, carrying 1 in the usual way to get

2 235

6 235

4 235

470

Now multiply the 235 of the second row by the 6 in the left hand column writing the number in the line below the 470 but moved one place to the right

2 235

6 235

4 235

470

1410

Now multiply the 235 of the third row by the 4 in the left hand column writing the number in the line below the 1410 but moved one place to the right

2 235

6 235

4 235

470

1410

940

Now add the three numbers below the line

2 235

6 235

4 235

470

1410

940

62040

The variants are first writing the second number on the right but with the order of the digits reversed as follows

235 4

235 6

235 2

940
1410
470

62040

The third variant just writes each number once but otherwise follows the second method

235

940 4
1410 6
470 2

62040

Another arithmetical result presented by Brahmagupta is his algorithm for computing square roots . This algorithm is discussed in [15] where it is shown to be equivalent to the Newton - Raphson iterative formula .

Brahmagupta developed some algebraic notation and presents methods to solve quardatic equations .He presents methods to solve indeterminate equations of the form $ax + c = by$. Majumdar in [17] writes: -

Brahmagupta perhaps used the method of continued fractions to find the integral solution of an indeterminate equation of the type $ax + c = by$.

In [17] Majumdar gives the original Sanskrit verses from Brahmagupta s Brahmasphuta siddhanta and their English translation with modern interpretation.

Brahmagupta also solves quadratic indeterminate equations of the type $ax^2 + c = y^2$ and $ax^2 - c = y^2$. For example he solves $8x^2 + 1 = y^2$ obtaining the solutions $(x, y) = (1, 3)$, $(6, 17)$, $(35, 99)$, $(204, 577)$, $(1189, 3363)$, …For the equation $11x^2 + 1 = y^2$ Brahmagupta obtained the solutions $(x, y) = (3, 10)$, $(161/5, 534/5)$, …He also solves $61x^2 + 1 = y^2$ which is particularly elegant having $x = 226153980$, $y = 1766319049$ as its smallest solution.

A example of the type of problems Brahmagupta poses and solves in the Brahmasphutasiddhanta is the following: -

Five hundred drammas were loaned at an unknown rate of interest, The interest on the money for four months was loaned to another at the same rate of interest and amounted in ten months to 78 drammas. Give the rate of interest.

Rules for summing series are also given. Brahmagupta gives the sum of the squares of the first n natural numbers as n

$(n+1)(2n+1)/6$ and the sum of the cubes of the first n natural numbers as $(n(n+1)/2)^2$. No proofs are given so we do not know how Brahmagupta discovered these formulas.

In the Brahmasphutasiddhanta Brahmagupta gave remarkable formulas for the area of a cyclic quadrilateral and for the lengths of the diagonals in terms of the sides. The only debatable point here is that Brahmagupta does not state that the formulas are only true for cyclic quadrilaterals so some historians claim it to be an error while others claim that he clearly meant the rules to apply only to cyclic quadrilaterals.

Much material in the Brahmasphutasiddhanta deals with solar and lunar eclipses, planetary conjunctions and positions of the planets. Brahmagupta believed in a static Earth and he gave the length of the year as 365 days 6 hours 5 minutes 19 seconds in the first work, changing the value to 365 days 6 hours 12 minutes 36 seconds in the second book the Khandakhadyaka. This second values is not, of course, an improvement on the first since the true length of the years if less than 365 days 6 hours. One has to wonder whether Brahmagupta s second value for the length of the year is taken from Aryabhata I since the two agree to within 6 seconds, yet are about 24 minutes out.

The Khandakhadyaka is in eight chapters again covering topics such as: the longitudes of the planets; the three problems of diurnal rotation; lunar eclipses; solar eclipses; risings and settings; the moon s crescent; and conjunctions of the planets. It contains an appendix which is some versions has only one chapter, in other versions has three.

Of particular interest to mathematics in this second work by Brahmagupta is the interpolation formula he uses to compute values of sines. This is studied in detail in [13] where it is shown to be a particular case up to second order of the more general Newton - Stirling interpolation formula.

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Augustus De Morgan

Born: 27 **June** 1806 **in Madura, Madras Presidency, India (now Madurai, Tamil Nadu, India)**

Died: 18 **March** 1871 **in London, England**

Augustus De Morgan s father John was a Lieutenant - Colonel who served in India. While he was stationed there his fifth child Augustus was born. Augustus lost the sight of his right eye shortly after birth and, when seven months old, returned to England with the family. John De Morgan died when Augustus was 10 years old.

At school De Morgan did not excel and, because of his physical disability: -

… he did not join in the sports of other boys, and he was even made the victim of cruel practical jokes by some schoolfellows.

De Morgan entered Trinity College Cambridge in 1823 at the age of 16 where he was taught by Peacock and Whewell - the three became lifelong friends. He received his BA but, because a theological test was required for the MA, something to which De Morgan strongly objected despite being a member of the Church of England, he could go no further at Cambridge being not eligible for a Fellowship without his MA.

In 1826 he returned to his home in London and entered Lincoln s Inn to study for the Bar. In 1827 (at the age of 21) he applied for the chair of mathematics in the newly founded University College London and, despite having no mathematical publications, he was appointed.

In 1828 De Morgan became the first professor of mathematics at University College. He gave his inaugural lecture On the study of mathematics. De Morgan was to resign his chair, on a matter of principle, is 1831. He was appointed to the chair again in 1836 and held it until 1866 when he was to resign for a second time, again on a matter of principle.

His book Elements of arithmetic (1830) was his second publication and was to see many editions.

In 1838 he defined and introduced the term ' mathematical induction ' putting a process that had been used without clarity on a rigorous basis. The term first appears in De Morgan s article Induction (Mathematics) in the Penny Cyclopedia. (Over the years he was to write 712 articles for the Penny Cyclopedia.) The Penny Cyclopedia was published by the Society for the Diffusion of Useful Knowledge, set up by the same reformers who founded London University, and that Society also published a famous work by De Morgan The Differential and Integral Calculus.

In 1849 he published Trigonometry and double algebra in which he gave a geometric interpretation of complex numbers.

He recognised the purely symbolic nature of algebra and he was aware of the existence of algebras other than ordinary algebra. He introduced De Morgan s laws and his greatest contribution is as a reformer of mathematical logic.

De Morgan corresponded with Charles Babbage and gave private tuition to Lady Lovelace who, it is claimed, wrote the first computer program for Babbage.

De Morgan also corresponded with Hamilton and, like Hamilton attempted to extend double algebra to three dimension. In a letter to Hamilton, De Morgan writes of his correspondence with Hamilton and William Hamilton. He writes: -

Be it known unto you that I have discovered that you and the other Sir W. H. are reciprocal polars with respect to me (intellectually and morally, for the Scottish baronet is a polar bear, and you, I was going to say, are a polar gentleman). When I send a bit of investigation to Edinburgh, the W. H. of that ilk says I took it from him. When I send you one, you take it from me, generalise it at a glance, bestow it thus generalised upon society at large, and make me the second discoverer of a known theorem.

In 1866 he was a co-founder of the London Mathematical Society and became its first president. De Morgan s son George, a very able mathematician, became its first secretary. In the same year De Morgan was elected a Fellow of the Royal Astronomical Society.

De Morgan was never a Fellow of the Royal Society as he refused to let his name be put forward. He also refused an honorary degree from the University of Edinburgh. He was described by Thomas Hirst thus:

A dry dogmatic pedant I fear is Mr De Morgan, notwithstanding his unquestioned ability.

Macfarlane remarks that: -

… De Morgan considered himself a Briton unat-

tached neither English, Scottish, Welsh or Irish.

He also says: -

> *He disliked the country and while his family enjoyed the seaside, and men of science were having a good time at a meeting of the British Association in the country he remained in the hot and dusty libraries of the metropolis.… He had no ideas or sympathies in common with the physical philosopher. His attitude was doubtless due to his physical infirmity, which prevented him from being either an observer or an experimenter. He never voted in an election, and he never visited the House of Commons, or the Tower, or Westminster Abbey.*

De Morgan was always interested in odd numerical facts and writing in 1864 he noted that he had the distinction of being x years old in the year x^2 (He was 43 in 1849). Anyone born in 1980 can claim the same distinction.

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Govindasvami

Born: bout 800 in India

Died: bout 860 in India

Govindasvami (or Govindasvamin) was an Indian mathematical astronomer whose most famous treatise was a commentary on the Mahabhaskariya of Bhaskara I.

Bhaskara I wrote the Mahabhaskariya in about 600 A.D. It is an eight-chapter work on Indian mathematical astronomy and includes topics which were fairly standard for such works at this time. It discussed topics such as the longitudes of the planets, conjunctions of the planets with each other and with bright stars, eclipses of the sun and the moon, risings and settings, and the lunar crescent.

Govindasvami wrote the Bhasya in about 830 which was a commentary on the Mahabhaskariya. In Govindasvami s commentary there appear many examples of using a place-value Sanskrit system of numerals. One of the most interesting aspects of the commentary, however, is Govindasvami s construction of a sine table.

Indian mathematicians and astronomers constructed sine table with great precision. They were used to calculate the positions of the planets as accurately as possible so had to be computed with high degrees of accuracy. Govindasvami considered the sexagesimal fractional parts of the twenty - four tabular sine differences from the Aryabhatiya. These lead to more correct sine values at intervals of $90° / 24 = 3°45'$. In the commentary Govindasvami found certain other empirical rules relating to computations of sine differences in the argumental range of 60 to 90 degrees. Both of the references [1] and [2] are concerned with the sine tables in Govindasvami s work.

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Harish - Chandra

Born: 11 Oct 1923 in Kanpur, Uttar Pradesh, India

Died: 16 Oct 1983 in Princeton, New Jersey, USA

Harish - Chandra attended school in Kanpur, then attended the University of Allahabad. Here he studied theoretical physics, this direction being the result of studying Principles of Quantum Mechanics by Dirac. He was awarded a master s degree in 1943 and then he went to Bangalore to work further on theoretical physics.

After a short while Harish - Chandra went to Cambridge where he studied for his Ph.D. under Dirac s supervision. During his time in Cambridge he moved away from physics and became more interested in mathematics. While at Cambridge he attended a lecture by Pauli and pointed out a mistake in Pauli s work. The two were to become life long friends. Harish-Chandra obtained his degree in 1947 and, the same year, he went to the USA.

Dirac visited Princeton for one year and Harish-Chandra worked as his assistant during this time. However he was greatly influenced by Weyl and Chevalley. The period 1950 to 1963 was his most productive and he spent these years at the Columbia University. During this time he worked on representations of semisimple Lie groups. Also during this period he had close contact with Weil.

In [4] Harish-Chandra is quoted as saying that he believed that his lack of background in mathematics was in a way responsible for the novelty of his work: -

I have often pondered over the roles of knowledge or experience, on the one hand, and imagination or intuition, on the other, in the process of discovery. I believe that there is a certain fundamental conflict between the two, and knowledge, by advocating caution, tends to inhibit the flight of imagination. Therefore, a certain naiveté, unburdened by conventional wisdom, can sometimes be a positive as-

set .

Harish - Chandra worked at the Institute for Advanced Study at Princeton from 1963 . He was appointed IBM - von Neumann Professor in 1968 .

He died of a heart attack at the end of a week long conference in Princeton, having earlier suffered from three - heart attacks .

Harish - Chandra received many awards in his career . He was a Fellow of the Royal Society of London and a Fellow of the National Academy of Sciences . He won the Cole prize from the American Mathematical Society in 1954 for his papers on representations of semisimple Lie algebras and groups, and particularly for his paper On some applications of the universal enveloping algebra of a semisimple Lie algebra which he had published in the Transactions of the American Mathematical Society in 1951 . In 1974, he received the Ramanujan Medal from Indian National Science Academy .

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Jagannatha Samrat

Born: bout 1690 in Amber (now Jaipur), India

Died: bout 1750 in India

Jagannatha had Jai Singh Sawai as his patron. Jai Singh Sawai was the ruler of Amber, now Jaipur, in eastern Rajasthan. He began his rule in 1699 and by clever use of tax rights on land that was rented by the state to an individual person he became the most important ruler in the region. His financial success let him finance the scholarly works of people such as Jagannatha. It is worth noting that Jai Singh s importance was recognised by Amber which was then called Jaipur in his honour.

Jai Singh ruled Amber throughout the period in which Jagannatha was producing his scientific work. He realised that the health of the country required Indian culture and science to be revitalised and returned to its position of leading importance which it had possessed. So Jai Singh employed Jagannatha to make Sanskrit translations of the important Greek scientific works which at that time were only available in Arabic translations.

Jagannatha translated Euclid s Elements from the Arabic translation by Nasir al - Din al - Tusi made nearly 500 years earlier. His Sanskrit version was called Rekhaganita and it was completed by 1727. We know this date since a copy was made by a scribe and he dated the start of his work as 1727.

Ptolemy s Almagest had been one of the works which Arabic scientists had studied intently and, in 1247, al - Tusi wrote Tahrir al - Majisti (Commentary on the Almagest) in which he introduced various trigonometrical techniques to calculate tables of sines. Jagannatha translated al - Tusi s Arabic version but he did more than this for he included in the same work, which he called Siddhantasamrat, his own comments on related work of other Arabic mathematical astronomers such as Ulugh Beg and al - Kashi.

It is clear from Jagannatha s work that he is working as one of a group of mathematicians and astronomers gathered by Jai Singh in his scheme to bring the best in scientific ideas from outside India to reinvigorate the scientific scene in India.

In [3] Gupta looks at the history of the result

$$\sin(\pi/10) = (5 - 1)/4$$

in Indian mathematics. The result appears for the first time in the work of Bhaskara II, but there were a number of interesting proofs of the result by later Indian mathematicians. One of the proofs presented by Gupta in [3] was by Jagannatha who gave a proof which was essentially geometric in nature but, interestingly, contained an analytic procedure in terms of trigonometric and algebraic steps.

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Jyesthadeva

Born: bout 1500 in Kerala, India
Died: bout 1575 in Kerala, India

Jyesthadeva lived on the southwest coast of India in the district of Kerala. He belonged to the Kerala school of mathematics built on the work of Madhava, Nilakantha Somayaji, Paramesvara and others.

Jyesthadeva wrote a famous text Yuktibhasa which he wrote in Malayalam, the regional language of Kerala. The work is a survey of Kerala mathematics and, very unusually for an Indian mathematical text, it contains proofs of the theorems and gives derivations of the rules it contains. It is one of the main astronomical and mathematical texts produced by the Kerala school. The work was based mainly on the Tantrasamgraha of Nilakantha.

The Yuktibhasa is a major treatise, half on astronomy and half on mathematics, written in 1501. The Tantrasamgraha on which it is based consists of 432 Sanskrit verses divided into 8 chapters, and it covers various aspects of Indian astronomy. It is based on the epicyclic and eccentric models of planetary motion. The first two chapters deal with the motions and longitudes of the planets. The third chapter Treatise on shadow deals with various problems related with the sun s position on the celestial sphere, including the relationships of its expressions in the three systems of coordinates, namely ecliptic, equatorial and horizontal coordinates.

The fourth and fifth chapters are Treatise on the lunar eclipse and On the solar eclipse and these two chapters treat various aspects of the eclipses of the sun and the moon. The sixth chapter is On vyatipata and deals with the complete deviation of the longitudes of the sun and the moon. The seventh chapter On visibility computation discusses the rising and setting of the moon and planets. The final chapter On elevation of the lunar cusps examines the size of the part of the moon which is illuminated by the sun and gives a graphical representation of it.

The Yuktibhasa is very important in terms of the mathematics Jyesthadeva presents. In particular he presents results discovered by Madhava and the treatise is an important source of the remarkable mathematical theorems which Madhava discovered. Written in about 1550, Jyesthadeva s commentary contained proofs of the earlier results by Madhava and Nilakantha which these earlier authors did not give. In [4] Gupta gives a translation of the text and this is also given in [2] and a number of other sources. Jyesthadeva describes Madhava s series as follows: -

The first term is the product of the given sine and radius of the desired arc divided by the cosine of the arc. The succeeding terms are obtained by a process of iteration when the first term is repeatedly multiplied by the square of the sine and divided by the square of the cosine. All the terms are then divided by the odd numbers 1, 3, 5, ⋯ *The arc is obtained by adding and subtracting respectively the terms of odd rank and those of even rank. It is laid down that the sine of the arc or that of its complement whichever is the smaller should be taken here as the given sine. Otherwise the terms obtained by this above iteration will not tend to the vanishing magnitude.*

This is a remarkable passage describing Madhava s series, but remember that even this passage by Jyesthadeva was written more than 100 years before James Gregory rediscovered this series expansion. To see how this description of the series fits with Gregory s series for arctan(x) see the biography of

Madhava. Other mathematical results presented by Jyesthadeva include topics studied by earlier Indian mathematicians such as integer solutions of systems of first degree equation solved by the kuttaka method, and rules of finding the sines and the cosines of the sum and difference of two angles.

Not only does the mathematics anticipate work by European mathematicians a century later, but the planetary theory presented by Jyesthadeva is similar to that adopted by Tycho Brahe.

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Kamalakara

Born: bout 1616 in Benares (now Varanasi), India
Died: bout 1700 in India

Kamalakara was an Indian astronomer and mathematician who came from a family of famous astronomers. Kamalakara s father was Nrsimha who was born in 1586. Two of Kamalakara s three brothers were also famous astronomer/ mathematicians, these being Divakara, who was the eldest of the brothers born in 1606, and Ranganatha who was younger than Kamalakara.

As was common throughout the classical period of Indian mathematics, members of the family acted as teachers to other family members. In particular Kamalakara was taught by his elder brother Divakara while Divakara himself had been taught by their uncle Siva. Pingree writes in [1]: -

> [*Kamalakara*] *combined traditional Indian astronomy with Aristotelian physics and Ptolemaic astronomy as presented by Islamic scientists* (*especially Ulugh Beg*). *Following his family s tradition he*

wrote a commentary, Manorama, on Ganesa s Grahalaghava and, like his father, Nrsimha, another commentary on the Suryasiddhanta, called the Vasanabhasya …

Kamalakara s most famous work, the Siddhanta - tattva - viveka, was commented on by Kamalakara himself . The work was completed in 1658 . It is a work of fifteen chapters covering standard topics for Indian astronomy texts at this time . It deals with the topics of: units of time measurement; mean motions of the planets; true longitudes of the planets; the three problems of diurnal rotation; diameters and distances of the planets; the earth s shadow; the moon s crescent; risings and settings; syzygies; lunar eclipses, solar eclipses; planetary transits across the sun s disk; the patas of the moon and sun; the "great problems"; and a final chapter which forms a conclusion .

The third chapter of the Siddhanta - tattva - viveka contains some of the most interesting mathematical results . In that chapter Kamalakara used the addition and subtraction theorems for the sine and the cosine to give trigonometric formulas for the sines and cosines of double, triple, quadruple and quintuple angles . In particular he gives formulas for $\sin(A/2)$ and $\sin(A/4)$ in terms of $\sin(A)$ and iterative formulas for $\sin(A/3)$ and $\sin(A/5)$. See for example [7] and [8] for a discussion of the details of Kamalakara s work in this area .

The Siddhanta - tattva - viveka is a Sanskrit text and in it Kamalakara makes frequent use of the place - value number system with Sanskrit numerals . This and many other aspects of the work are discussed in [3] .

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Katyayana

Born: bout 200 BC in India

Died: bout 200 BC in India

We cannot attempt to write a biography of Katyayana since essentially nothing is known of him except that he was the author of a Sulbasutra which is much later than the Sulbasutras of Baudhayana and Apastamba. It would also be fair to say that Katyayana s Sulbasutra is the least interesting from a mathematical point of view of the three best known Sulbasutras. It adds very little to that of Apastamba written several hundreds of years earlier. We do not know Katyayana s dates accurately enough to even guess at a life span for him, which is why we have given the same approximate birth year as death year.

Katyayana was neither a mathematician in the sense that we would understand it today, nor a scribe who simply copied manuscripts like Ahmes. He would certainly have been a man of very considerable learning but probably not interested in mathematics for its own sake, merely interested in using it for religious purposes. Undoubtedly he wrote the Sulbasutra to provide rules for religious rites and to improve and expand on the rules which had been given by his predecessors. Katyayana would have been a priest instructing the people in the ways of conducting the religious rites he describes.

Katyayana lived in a period when the religious rites that the Sulbasutras were written to support were becoming less influential. People were turning to other religions and perhaps this lack of vigour in the religion at this time partly explains why several hundreds of years after Apastamba Katyayana adds little of importance to the Sulbasutra which he wrote.

See the article Indian Sulbasutras for more information on the Sulbasutras in general and the mathematical results which they contain.

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Lalla

Born: bout 720 in India

Died: bout 790 in India

Lalla s father was Trivikrama Bhatta and Trivikrama s father, Lalla s paternal grandfather, was named Samba. Lalla was an Indian astronomer and mathematician who followed the tradition of Aryabhata I. Lalla s most famous work was entitled Shishyadhividdhidatantra. This major treatise was in two volumes. The first volume, On the computation of the positions of the planets, was in thirteen chapters and covered topics such as: mean longitudes of the planets; true longitudes of the planets; the three problems of diurnal rotation; lunar eclipses; solar eclipses; syzygies; risings and settings; the shadow of the moon; the moon s crescent; conjunctions of the planets with each other; conjunctions of the planets with the fixed stars; the patas of the moon and sun, and a final chapter in the first volume which forms a conclusion.

The second volume was On the sphere. In this volume Lalla examined topics such as: graphical representation; the celestial sphere; the principle of mean motion; the terrestrial sphere; motions and stations of the planets; geography; erroneous knowledge; instruments; and finally selected problems.

In Shishyadhividdhidatantra Lalla uses Sanskrit numerical symbols. Ifrah writes in [2]: -

> …*over the centuries, Sanskrit has lent itself admirably to the rules of prosody and versification. This explains why Indian astronomers [like Lalla] favoured the use of Sanskrit numerical symbols, based on a complex symbolism which was extraordinarily fertile and sophisticated, possessing as it did an almost limitless choice of synonyms.*

Despite writing the most famous treatise giving the views of Aryabhata I, Lalla did not accept his theory given in the Aryabhatiya that the earth rotated. Lalla argues in his commentary, like many other Indian astronomers before him such as Varahamihira and Brahmagupta, that if the earth rotated then the speed would have to be such that one would have to ask how do the bees or birds flying in the sky come back to their nests? In fact Lalla misinterpreted some of Aryabhata I s statements about the rotating earth. One has to assume that the idea appeared so impossible to him that he just could not appreciate

Aryabhata I s arguments. As Chatterjee writes in [3], Lalla in his commentary: -

> … *did not interpret the relevant verses in the way meant by Aryabhata I.*

Astrology at this time was based on astronomical tables and often the horoscopes allow one to identify the tables used. Some Arabic horoscopes were based on astronomical tables calculated in India. The most frequently used tables were by Aryabhata I. Lalla improved on these tables and he produced a set of corrections for the Moon s longitude. One aspect of Aryabhata I s work which Lalla did follow was his value of π. Lalla uses $\pi = 62832/20000$, i.e. $\pi = 3.1416$ which is a value correct to the fourth decimal place.

Lalla also wrote a commentary on Khandakhadyaka, a work of Brahmagupta. Lalla s commentary has not survived but there is another work on astrology by Lalla which has survived, namely the Jyotisaratnakosa. This was a very popular work which was the main one on the subject in India for around 300 years.

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Madhava of Sangamagramma

Born: 1350 **in Sangamagramma (near Cochin), Kerala, India**
Died: 1425 **in India**

Madhava of Sangamagramma was born near Cochin on the coast in the Kerala state in southwestern India. It is only due to research into Keralese mathematics over the last twenty-five years that the remarkable contributions of Madhava have come to light. In [10] Rajagopal and Rangachari put his achievement into context when they write: -

> [*Madhava*] *took the decisive step onwards from the finite procedures of ancient mathematics to treat their limit-passage to infinity, which is the kernel of modern classical analysis.*

Now all the mathematical writings of Madhava have been lost although some of his texts on astronomy have survived. However his brilliant work in mathematics has been largely discovered by the reports of other Keralese mathematicians such as Nilakantha who lived about 100 years later.

Madhava discovered the series equivalent to the Maclaurin expansions of sinx, cosx, and arctanx around 1400, which is over two hundred years before they were rediscovered in Europe. Details appear in a number of works written by his followers such as Mahajyanayana prakara which means Method of computing the great sines. In fact this work had been claimed by some historians such as Sarma (see for example [2]) to be by Madhava himself but this seems highly unlikely and it is now accepted by most historians to be a 16th century work by a follower of Madhava. This is discussed in detail in [4].

Jyesthadeva wrote Yukti - Bhasa in Malayalam, the regional language of Kerala, around 1550. In [9] Gupta gives a translation of the text and this is also given in [2] and a number of other sources. Jyesthadeva describes Madhava s series as follows: -

> *The first term is the product of the given sine and radius of the desired arc divided by the cosine of the arc. The succeeding terms are obtained by a process of iteration when the first term is repeatedly multiplied by the square of the sine and divided by the square of the cosine. All the terms are then divided by the odd numbers* 1, 3, 5, ⋯ *The arc is obtained by adding and subtracting respectively the*

terms of odd rank and those of even rank. It is laid down that the sine of the arc or that of its complement whichever is the smaller should be taken here as the given sine. Otherwise the terms obtained by this above iteration will not tend to the vanishing magnitude.

This is a remarkable passage describing Madhava s series, but remember that even this passage by Jyesthadeva was written more than 100 years before James Gregory rediscovered this series expansion. Perhaps we should write down in modern symbols exactly what the series is that Madhava has found. The first thing to note is that the Indian meaning for sine of q would be written in our notation as r sin q and the Indian cosine of would be r cos q in our notation, where r is the radius. Thus the series is

$$rq = r(r\sin q)/1(r\cos q) - r(r\sin q)^3/3r(r\cos q)^3 + r(r\sin q)^5/5r(r\cos q)^5 - r(r\sin q)^7/7r(r\cos q)^7 + \cdots$$

putting tan = sin/ cos and cancelling r gives

$$q = \tan q - (\tan^3 q)/3 + (\tan^5 q)/5 - \cdots$$

which is equivalent to Gregory s series

$$\tan^{-1} q = q - q^3/3 + q^5/5 - \cdots$$

Now Madhava put $q = \pi/4$ into his series to obtain

$$\pi/4 = 1 - 1/3 + 1/5 - \cdots$$

and he also put $q = \pi/6$ into his series to obtain

$$\pi = 12(1 - 1/(3\times 3) + 1/(5\times 3^2) - 1/(7\times 3^3) + \cdots$$

We know that Madhava obtained an approximation for π correct to 11 decimal places when he gave

$$\pi = 3.14159265359$$

which can be obtained from the last of Madhava s series above by taking 21 terms. In [5] Gupta gives a translation of the Sanskrit text giving Madhava s approximation of π correct to 11 places.

Perhaps even more impressive is the fact that Madhava gave a remainder term for his series which improved the approximation. He improved the approximation of the series for $\pi/4$ by adding a correction term R_n to obtain

$$\pi/4 = 1 - 1/3 + 1/5 - \cdots 1/(2n - 1) \pm R_n$$

Madhava gave three forms of R_n which improved the approximation, namely

$R_n = 1/(4n)$ or

$R_n = n/(4n^2 + 1)$ or

$R_n = (n^2 + 1)/(4n^3 + 5n)$.

There has been a lot of work done in trying to reconstruct how Madhava might have found his correction terms. The most convincing is that they come as the first three convergents of a continued fraction which can itself be derived from the standard

Indian approximation to π namely 62832/ 20000 .

Madhava also gave a table of almost accurate values of half - sine chords for twenty - four arcs drawn at equal intervals in a quarter of a given circle . It is thought that the way that he found these highly accurate tables was to use the equivalent of the series expansions

$$\sin q = q - q^3/3! + q^5/5! - \cdots$$

$$\cos q = 1 - q^2/2! + q^4/4! - \cdots$$

Jyesthadeva in Yukti - Bhasa gave an explanation of how Madhava found his series expansions around 1400 which are equivalent to these modern versions rediscovered by Newton around 1676 . Historians have claimed that the method used by Madhava amounts to term by term integration .

Rajagopal s claim that Madhava took the decisive step towards modern classical analysis seems very fair given his remarkable achievements . In the same vein Joseph writes in [1]: -

> *We may consider Madhava to have been the founder of mathematical analysis . Some of his discoveries in this field show him to have possessed extraordinary intuition, making him almost the equal of the more recent intuitive genius Srinivasa Ramanujan, who spent his childhood and youth at Kumbakonam, not far from Madhava s birthplace .*

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Mahavira

Born: bout 800 in possibly Mysore, India

Died: bout 870 in India

Mahavira (or Mahaviracharya meaning Mahavira the Teacher) was of the Jaina religion and was familiar with Jaina mathematics. He worked in Mysore in southern Indian where he was a member of a school of mathematics. If he was not born in Mysore then it is very likely that he was born close to this town in the same region of India. We have essentially no other biographical details although we can gain just a little of his personality from the acknowledgement he gives in the introduction to his only known work, see below. However Jain in [10] mentions six other works which he credits to Mahavira and he emphasises the need for further research into identifying the complete list of his works.

The only known book by Mahavira is Ganita Sara Samgraha, dated 850 AD, which was designed as an updating of Brahmagupta s book. Filliozat writes [6]: -

This book deals with the teaching of Brahmagupta but contains both simplifications and additional information. ··· Although like all Indian versified texts, it is extremely condensed, this work, from a pedagogical point of view, has a significant advantage over earlier texts.

It consisted of nine chapters and included all mathematical knowledge of mid - ninth century India. It provides us with the bulk of knowledge which we have of Jaina mathematics and it can be seen as in some sense providing an account of the work of those who developed this mathematics. There were many Indian mathematicians before the time of Mahavira but, perhaps surprisingly, their work on mathematics is always contained in texts which discuss other topics such as astronomy. The Ganita Sara Samgraha by Mahavira is the earliest Indian text which we possess which is devoted entirely to mathematics.

In the introduction to the work Mahavira paid tribute to the mathematicians whose work formed the basis of his book. These mathematicians included Aryabhata I, Bhaskara I, and Brahmagupta. Mahavira writes: -

With the help of the accomplished holy sages, who are worthy to be worshipped by the lords of the world ··· I glean from the great ocean of the knowl-

edge of numbers a little of its essence, in the manner in which gems are picked from the sea, gold from the stony rock and the pearl from the oyster shell; and I give out according to the power of my intelligence, the Sara Samgraha, a small work on arithmetic, which is however not small in importance.

The nine chapters of the Ganita Sara Samgraha are:

1. Terminology
2. Arithmetical operations
3. Operations involving fractions
4. Miscellaneous operations
5. Operations involving the rule of three
6. Mixed operations
7. Operations relating to the calculations of areas
8. Operations relating to excavations
9. Operations relating to shadows

Throughout the work a place - value system with nine numerals is used or sometimes Sanskrit numeral symbols are used. Of interest in Chapter 1 regarding the development of a place - value number system is Mahavira s description of the number 12345654321 which he obtains after a calculation. He describes the number as: -

…beginning with one which then grows until it

reaches six, then decreases in reverse order.

Notice that this wording makes sense to us using a place - value system but would not make sense in other systems. It is a clear indication that Mahavira is at home with the place - value number system.

Among topics Mahavira discussed in his treatise was operations with fractions including methods to decompose integers and fractions into unit fractions. For example

$2/17 = 1/12 + 1/51 + 1/68$.

He examined methods of squaring numbers which, although a special case of multiplying two numbers, can be computed using special methods. He also discussed integer solutions of first degree indeterminate equation by a method called kuttaka. The kuttaka (or the "pulveriser") method is based on the use of the Euclidean algorithm but the method of solution also resembles the continued fraction process of Euler given in 1764. The work kuttaka, which occurs in many of the treatises of Indian mathematicians of the classical period, has taken on the more general meaning of "algebra".

An example of a problem given in the Ganita Sara Samgraha which leads to indeterminate linear equations is the following:

Three merchants find a purse lying in the road.

One merchant says "If I keep the purse, I shall have twice as much money as the two of you together". "Give me the purse and I shall have three times as much" said the second merchant. The third merchant said "I shall be much better off than either of you if I keep the purse, I shall have five times as much as the two of you together". How much money is in the purse? How much money does each merchant have?

If the first merchant has x, the second y, the third z and p is the amount in the purse then

$p + x = 2(y + z)$, $p + y = 3(x + z)$, $p + z = 5(x + y)$.

There is no unique solution but the smallest solution in positive integers is $p = 15$, $x = 1$, $y = 3$, $z = 5$. Any solution in positive integers is a multiple of this solution as Mahavira claims.

Mahavira gave special rules for the use of permutations and combinations which was a topic of special interest in Jaina mathematics. He also described a process for calculating the volume of a sphere and one for calculating the cube root of a number. He looked at some geometrical results including right - angled triangles with rational sides, see for example [4].

Mahavira also attempts to solve certain mathematical problems which had not been studied by other Indian mathematicians. For example, he gave an approximate formula for the area and the perimeter of an ellipse. In [8] Hayashi writes: -

> *The formulas for a conch - like figure have so far been found only in the works of Mahavira and Narayana.*

Now it is reasonable to ask what a" conch - like figure" is. It is two unequal semicircles (with diameters AB and BC) stuck together along their diameters. Although it might be reasonable to suppose that the perimeter might be obtained by considering the semicircles, Hayashi claims that the formulas obtained: -

> *… were most probably obtained not from the two semicircles AB and BC.*

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Mahendra Suri

Born: bout 1340 in Western India

Died: bout 1410 in India

Mahendra Suri was a Jain. Jainism began around the sixth century BC and the religion had a strong influence on mathematics particularly in the last couple of centuries BC. By the time of Mahendra Suri, however, Jainism had lost support as a national religion and was much less vigorous. It had been influenced by Islam and in particular Islamic astronomy came to form a part of the background. However, Pingree in [4] writes that this filtering of Islamic astronomy into Indian culture was: -

> *…not allowed to affect in any way the structure of the traditional science.*

Mahendra Suri was a pupil of Madana Suri. He is famed as the first person to write a Sanskrit treatise on the astrolabe. Ohashi writes in [3] of the early history of the astrolabe in the

Delhi Sultanate in India: -

The astrolabe was introduced into India at the time of Firuz Shah Tughluq (reign AD 1351 - 88), *and Mahendra Suri wrote the first Sanskrit treatise on the astrolabe entitled Yantraraja (AD* 1370).

The Delhi Sultanate was established around 1200 and from that time on Muslim culture flourished in India. The ideas of Islamic astronomy began to appear in works in the Sanskrit language and it is the Islamic ideas on the astrolabe which Mahendra Suri wrote on in his famous text. It is clear from the various references in the text and also from the particular values that Mahendra Suri uses for the angle of the ecliptic etc. that his work is based on Islamic rather than traditional Indian astronomy works.

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Manava

Born: bout 750 BC in India

Died: bout 750 BC in India

Manava was the author of one of the Sulbasutras. The Manava Sulbasutra is not the oldest (the one by Baudhayana is older) nor is it one of the most important, there being at least three Sulbasutras which are considered more important. We do not know Manava s dates accurately enough to even guess at a life span for him, which is why we have given the same approximate birth year as death year. Historians disagree on 750 BC, and some would put this Sulbasutra later by one hundred or more years.

Manava would have not have been a mathematician in the sense that we would understand it today. Nor was he a scribe who simply copied manuscripts like Ahmes. He would certainly have been a man of very considerable learning but probably not interested in mathematics for its own sake, merely interested in using it for religious purposes. Undoubtedly he wrote the Sulbasutra to provide rules for religious rites and it would appear an almost certainty that Manava himself would be a Vedic priest.

The mathematics given in the Sulbasutras is there to enable accurate construction of altars needed for sacrifices. It is clear from the writing that Manava, as well as being a priest, must have been a skilled craftsman.

Manava s Sulbasutra, like all the Sulbasutras, contained approximate constructions of circles from rectangles, and squares from circles, which can be thought of as giving approximate values of π. There appear therefore different values of π throughout the Sulbasutra, essentially every construction involving circles leads to a different such approximation. The paper [1] is concerned with an interpretation of verses 11.14 and 11.15 of Manava s work which give $\pi = 25/8 = 3.125$.

See the article Indian Sulbasutras for more information on the Sulbasutras in general and the mathematical results which they contain.

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Narayana Pandit

Born: bout 1340 in India

Died: bout 1400 in India

Narayana was the son of Nrsimha (sometimes written Narasimha). We know that he wrote his most famous work Ganita Kaumudi on arithmetic in 1356 but little else is known of him. His mathematical writings show that he was strongly influenced by Bhaskara II and he wrote a commentary on the Lilavati of Bhaskara II called Karmapradipika. Some historians dispute that Narayana is the author of this commentary which they attribute to Madhava.

In the Ganita Kaumudi Narayana considers the mathematical operation on numbers. Like many other Indian writers of arithmetics before him he considered an algorithm for multiplying numbers and he then looked at the special case of squaring numbers. One of the unusual features of Narayana s work Karmapradipika is that he gave seven methods of squaring numbers which are not found in the work of other Indian mathematicians.

He discussed another standard topic for Indian mathematicians namely that of finding triangles whose sides had integral values. In particular he gave a rule of finding integral triangles whose sides differ by one unit of length and which contain a pair of right-angled triangles having integral sides with a common integral height. In terms of geometry Narayana gave a rule for a segment of a circle. Narayana [4]: -

> *… derived his rule for a segment of a circle from Mahavira s rule for an 'elongated circle' or an ellipse-like figure.*

Narayana also gave a rule to calculate approximate values of a square root. He did this by using an indeterminate equation of the second order, $Nx^2 + 1 = y^2$, where N is the number whose square root is to be calculated. If x and y are a pair of roots of this equation with $x < y$ then N is approximately equal to y/x. To illustrate this method Narayana takes $N = 10$. He then finds the solutions $x = 6$, $y = 19$ which give the approximation $19/6 = 3.1666666666666667$, which is correct to 2 decimal places. Narayana then gives the solutions $x = 228$, $y = 721$ which give the approximation $721/228 = 3.1622807017543859649$, correct to four places. Finally Narayana gives the pair of solutions $x = 8658$, $y = 227379$ which give the approximation $227379/8658 = 3.1622776622776622777$, correct to eight decimal places. Note for comparison that 10 is, correct to 20 places, 3.16227766016837933320. See [3] for more information.

The thirteenth chapter of Ganita Kaumudi was called Net of Numbers and was devoted to number sequences. For example, he discussed some problems concerning arithmetic progressions.

The fourteenth chapter (which is the last one) of Naryana s Ganita Kaumudi contains a detailed discussion of magic squares and similar figures. Narayana gave the rules for the formation of doubly even, even and odd perfect magic squares along with magic triangles, rectangles and circles. He used formulas and rules for the relations between magic squares and arithmetic series. He gave methods for finding "the horizontal difference" and the first term of a magic square whose square s constant and the number of terms are given and he also gave rules for finding "the vertical difference" in the case where this information is given.

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Nilakantha Somayaji

Born: 14 June 1444 in Trkkantiyur (near Tirur), Kerala, India

Died: 1544 in India

Nilakantha was born into a Namputiri Brahmin family which came from South Malabar in Kerala. The Nambudiri is the main caste of Kerala. It is an orthodox caste whose members consider themselves descendants of the ancient Vedic religion.

He was born in a house called Kelallur which it is claimed coincides with the present Etamana in the village of Trkkantiyur near Tirur in south India. His father was Jatavedas and the family belonged to the Gargya gotra, which was a Indian caste that prohibits marriage to anyone outside the caste. The family followed the Ashvalayana sutra which was a manual of sacrificial ceremonies in the Rigveda, a collection of Vedic hymns. He worshipped the personified deity Soma who was the " master of plants" and the healer of disease. This explains the name Somayaji which means he was from a family qualified to conduct the Soma ritual .

Now Nilakantha studied astronomy and Vedanta, one of the six orthodox systems of Indian Hindu philosophy, under the teacher Ravi . He was also taught by Damodra who was the son of Paramesvara . Paramesvara was a famous Indian astronomer and Damodra followed his father s teachings . This led Nilakantha also to become a follower of Paramesvara . A number of texts on mathematical astronomy written by Nilakantha have survived . In all he wrote about ten treatises on astronomy .

The Tantrasamgraha is his major astronomy treatise written in 1501 . It consists of 432 Sanskrit verses divided into 8 chapters, and it covers various aspects of Indian astronomy . It is based on the epicyclic and eccentric models of planetary motion . The first two chapters deal with the motions and longitudes of the planets . The third chapter Treatise on shadow deals with various problems related with the sun s position on the celestial sphere, including the relationships of its expressions in the three systems of coordinates, namely ecliptic, equatorial and horizontal coordinates .

The fourth and fifth chapters are Treatise on the lunar eclipse and On the solar eclipse and these two chapters treat various aspects of the eclipses of the sun and the moon. The sixth chapter is On vyatipata and deals with the complete deviation of the longitudes of the sun and the moon. The seventh chapter On visibility computation discusses the rising and setting of the moon and planets. The final chapter On elevation of the lunar cusps examines the size of the part of the moon which is illuminated by the sun and gives a graphical representation of it.

The Tantrasamgraha is very important in terms of the mathematics Nilakantha uses. In particular he uses results discovered by Madhava and it is an important source of the remarkable mathematical results which he discovered. However, Nilakantha does not just use Madhava s results, he extends them and improves them. An anonymous commentary entitled Tantrasangraha - vakhya appeared and, somewhat later in about 1550, Jyesthadeva published a commentary entitled Yuktibhasa that contained proofs of the earlier results by Madhava and Nilakantha. This is quite unusual for an Indian text in giving mathematical proofs.

The series $\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \cdots$ is a special case of the series representation for arctan, namely

$$\tan^{-1} x = x - x^3/3 + x^5/5 - x^7/7 + \cdots$$

It is well - known that one simply puts $x = 1$ to obtain the series for $\pi/4$. The author of [4] reports on the appearance of these series in the work of Leibniz and James Gregory from the 1670s. The contributions of the twom European mathematicians to this series are well known but in [4] the results on this series in the work of Madhava nearly three hundred years earlier as presented by Nilakantha in the Tantrasamgraha is also discussed.

Nilakantha derived the series expansion

$$\tan^{-1} x = x - x^3/3 + x^5/5 - x^7/7 + \cdots$$

by obtaining an approximate expression for an arc of the circumference of a circle and then considering the limit. An interesting feature of his work was his introduction of several additional series for $\pi/4$ that converged more rapidly than

$$\pi/4 = 1 - 1/3 + 1/5 - 1/7 + \cdots.$$

The author of [4] provides a reconstruction of how he may have arrived at these results based on the assumption that he possessed a certain continued fraction representation for the tail series

$$1/(n+2) - 1/(n+4) + 1/(n+6) - 1/(n+8) + \cdots$$

The Tantrasamgraha is not the only work of Nilakantha of which we have the text. He also wrote Golasara which is written in fifty - six Sanskrit verses and shows how mathematical computations are used to calculate astronomical data. The Siddhanta Darpana is written in thirty - two Sanskrit verses and describes a planetary model. The Candracchayaganita is written in thirty - one Sanskrit verses and explains the computational methods used to calculate the moon s zenith distance.

The head of the Nambudiri caste in Nilakantha s time was Netranarayana and he became Nilakantha s patron for another of his major works, namely the Aryabhatiyabhasya which is a commentary on the Aryabhatiya of Aryabhata I. In this work Nilakantha refers to two eclipses which he observed, the first on 6 March 1467 and the second on 28 July 1501 at Anantaksetra. Nilakantha also refers in the Aryabhatiyabhasya to other works which he wrote such as the Grahanirnaya on eclipses which have not survived.

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Panini

Born: bout 520 BC in Shalatula (near Attock), now Pakistan

Died: bout 460 BC in India

Panini was born in Shalatula, a town near to Attock on the Indus river in present day Pakistan. The dates given for Panini are pure guesses. Experts give dates in the 4th, 5th, 6th and 7th century BC and there is also no agreement among historians about the extent of the work which he undertook. What is in little doubt is that, given the period in which he worked, he is one of the most innovative people in the whole development of knowledge. We will say a little more below about how historians have gone about trying to pinpoint the date when Panini lived.

Panini was a Sanskrit grammarian who gave a comprehensive and scientific theory of phonetics, phonology, and morphology. Sanskrit was the classical literary language of the Indian Hindus and Panini is considered the founder of the language and literature. It is interesting to note that the word "Sanskrit" means "complete" or "perfect" and it was thought of as the divine language, or language of the gods.

A treatise called Astadhyayi (or Astaka) is Panini s major work. It consists of eight chapters, each subdivided into quarter chapters. In this work Panini distinguishes between the language of sacred texts and the usual language of communication. Panini gives formal production rules and definitions to describe Sanskrit grammar. Starting with about 1700 basic elements like nouns, verbs, vowels, consonants he put them into classes. The construction of sentences, compound nouns etc. is explained as ordered rules operating on underlying structures in a manner similar to modern theory. In many ways Panini s constructions are similar to the way that a mathematical function is defined today. Joseph writes in [2]: -

> [*Sanskrit s*] *potential for scientific use was greatly enhanced as a result of the thorough systematisation of its grammar by Panini. … On the basis of just under 4000 sutras* [*rules expressed as aphorisms*], *he built virtually the whole structure of the Sanskrit language, whose general 'shape' hardly changed for the next two thousand years. … An in-*

direct consequence of Panini s efforts to increase the linguistic facility of Sanskrit soon became apparent in the character of scientific and mathematical literature. This may be brought out by comparing the grammar of Sanskrit with the geometry of Euclid - a particularly apposite comparison since, whereas mathematics grew out of philosophy in ancient Greece, it was … partly an outcome of linguistic developments in India.

Joseph goes on to make a convincing argument for the algebraic nature of Indian mathematics arising as a consequence of the structure of the Sanskrit language. In particular he suggests that algebraic reasoning, the Indian way of representing numbers by words, and ultimately the development of modern number systems in India, are linked through the structure of language.

Panini should be thought of as the forerunner of the modern formal language theory used to specify computer languages. The Backus Normal Form was discovered independently by John Backus in 1959, but Panini s notation is equivalent in its power to that of Backus and has many similar properties. It is remarkable to think that concepts which are fundamental to today s theoretical computer science should have their origin with an Indian genius around 2500 years ago.

At the beginning of this article we mentioned that certain concepts had been attributed to Panini by certain historians which others dispute. One such theory was put forward by B Indraji in 1876. He claimed that the Brahmi numerals developed out of using letters or syllables as numerals. Then he put the finishing touches to the theory by suggesting that Panini in the eighth century BC (earlier than most historians place Panini) was the first to come up with the idea of using letters of the alphabet to represent numbers.

There are a number of pieces of evidence to support Indraji s theory that the Brahmi numerals developed from letters or syllables. However it is not totally convincing since, to quote one example, the symbols for 1, 2 and 3 clearly do not come from letters but from one, two and three lines respectively. Even if one accepts the link between the numerals and the letters, making Panini the originator of this idea would seem to have no more behind it than knowing that Panini was one of the most innovative geniuses that world has known so it is not unreasonable to believe that he might have made this step too.

There are other works which are closely associated with the Astadhyayi which some historians attribute to Panini, others attribute to authors before Panini, others attribute to authors after Panini. This is an area where there are many theories but few, if any, hard facts.

We also promised to return to a discussion of Panini s dates. There has been no lack of work on this topic so the fact that there are theories which span several hundreds of years is not the result of lack of effort, rather an indication of the difficulty of the topic. The usual way to date such texts would be to examine which authors are referred to and which authors refer to the work. One can use this technique and see who Panini mentions.

There are ten scholars mentioned by Panini and we must assume from the context that these ten have all contributed to the study of Sanskrit grammar. This in itself, of course, indicates that Panini was not a solitary genius but, like Newton, had "stood on the shoulders of giants". Panini must have lived later than these ten but this is absolutely no help in providing dates since we have absolutely no knowledge of when any of these ten lived.

What other internal evidence is there to use? Well of course Panini uses many phrases to illustrate his grammar any these have been examined meticulously to see if anything is contained there to indicate a date. To give an example of what we mean: if we were to pick up a text which contained as an example "I take the train to work every day" we would know that it had to have been written after railways became common. Let us illustrate with two actual examples from the Astadhyayi which have been the subject of much study. The first is an attempt to see whether there is evidence of Greek influence. Would it be possible to find evidence which would mean that the text had to have been written after the conquests of Alexander the Great? There is a little evidence of Greek influence, but there was Greek influence on this north east part of the Indian subcontinent before the time of Alexander. Nothing conclusive has been identified.

Another angle is to examine a reference Panini makes to nuns. Some argue that these must be Buddhist nuns and therefore the work must have been written after Buddha. A nice argument but there is a counter argument which says that there were Jaina nuns before the time of Buddha and Panini s reference could equally well be to them. Again the evidence is inconclusive.

There are references by others to Panini. However it would appear that the Panini to whom most refer is a poet and although some argue that these are the same person, most historians agree that the linguist and the poet are two different people. Again this is inconclusive evidence.

Let us end with an evaluation of Panini s contribution by Cardona in [1]: -

> *Panini s grammar has been evaluated from various points of view. After all these different evaluations, I think that the grammar merits asserting …that it is one of the greatest monuments of human intelligence.*

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Paramesvara

Born: bout 1370 in Alattur, Kerala, India

Died: bout 1460 in India

Paramesvara was an Indian astronomer and mathematician who wrote many commentaries on earlier works as well as making many observations. Although his father has not been identified, we know that Paramesvara was born into a Namputiri Brahmana family who were astrologers and astronomers. The family home was Vatasseri (sometimes called Vatasreni) in the village of Alattur. This village was in Kerala and Paramesvara himself gives its coordinates with respect to Ujjain. This puts it at latitude 10°51′north. It is on the north bank of the river Nila at its mouth.

From Paramesvara s writing we know that Rudra was his teacher, and Nilakantha, who knew Paramesvara personally, tells us that Paramesvara s teachers included Madhava and Narayana. We can be fairly confident that the dates we have given for Paramesvara are roughly correct since he made eclipe observations over a period of 55 years. We will say a little more about these observations below. He played an important part in the remarkable developments in mathematics which took place in Kerala in the late 14th and early part of the 15th century.

The commentaries by Paramesvara on a number of works have been published. For example the Karmadipika is a commentary on the Mahabhaskariyam, an astronomical and mathematical work by Bhaskara I, and its text is given in [3]. In [2] the text of Paramesvara s commentary on the Laghubhaskariyam of Bhaskara I is given. Munjala wrote the astronomical work Laghumanasam in the year 932 and Paramesvara wrote a commentary (see [4]). It is a work containing typical topics for Indian mathematical astronomy works of this period: the mean motions of the heavenly bodies; the true motions of the heavenly bodies; miscellaneous mathematical rules; the systems of coordinates, direction, place and time; eclipses of the sun and the moon; and the operation for apparent longitude.

Aryabhata gave a rule for determining the height of a pole from the lengths of its shadows in the Aryabhatiya. Paramesvara gave several illustrative examples of the method in his commentary on the Aryabhatiya.

Like many mathematicians from Kerala, Madhava clearly had a very strong influence on Paramesvara. One can see throughout his work that it is teachings by Madhava which direct much of Paramesvara s mathematical ideas. One of Paramesvara s most remarkable mathematical discoveries, no doubt influenced by Madhava, was a version of the mean value theorem. He states the theorem in his commentary Lilavati Bhasya on Bhaskara II s Lilavati. There are other examples of versions of the mean value theorem in Paramesvara s work which we now consider.

The Siddhantadipika by Paramesvara is a commentary on the commentary of Govindasvami on Bhaskara I s Mahabhaskariya. Paramesvara gives some of his eclipse observations in this work including one made at Navaksetra in 1422 and two made at Gokarna in 1425 and 1430. This work also contains a mean value type formula for inverse interpolation of the sine. It presents a one - point iterative technique for calculating the sine of a given angle. In the Siddhantadipika Paramesvara also gives a more efficient approximation that works using a two - point iterative algorithm which turns out to be essentially the same as the modern secant method. See [8] and [9] for further details.

The expression for the radius of the circle in which a cyclic quadrilateral is inscribed, given in terms of the sides of the quadrilateral, is usually attributed to Lhuilier in 1782. However Paramesvara described the rule 350 years earlier. If the sides of the cyclic quadrilateral are a, b, c and d then the radius r of the circumscribed circle was given by Paramesvara as:

$r^2 = x/y$ where

$x = (ab + cd)(ac + bd)(ad + bc)$

and $y = (a + b + c - d)(b + c + d - a)(c + d + a - b)(d + a + b - c)$.

The original text by Paramesvara describing the rule is given in [7].

Paramesvara made a series of eclipse observations between 1393 and 1432 which we have referred to above. The last observation which we know he made was in 1445 but Nilakantha quotes a verse by Paramesvara in which he claims to have made observations spanning 55 years. The known observatons by Paramesvara do not quite square with this statement, there being a discrepancy of three years. Although we do not know when Paramesvara died we do know, again from Nilakantha, that the two knew each other personally. Since we have a definite date for Nilakantha s birth of 1444 it is hard to believe that Paramesvara died before 1460 .

Using his observations, Paramesvara made revisions of the planetary parameters and, like many other Indian astronomers, he constantly attempted to compare the theoretically computed positions of the planets with those which he actually observed . He was keen to improve the theoretical model to bring it into as close an agreement with observations as possible .

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Vijay Kumar Patodi

Born: 12 **March** 1945 **in Guna, Madhya Pradesh, India**

Died: 21 **Dec** 1976 **in Bombay, India**

Vijay Patodi attended the Government Higher Secondary School in Guna before entering Vikram University in Ujjain. After obtaining his B.Sc. degree from Vikram University, Patodi moved to Banares Hindu University were he studied for his Master s Degree in Mathematics. This was awarded in 1966 and Patodi then spent a year at the Centre for Advanced Study at the University of Bombay.

In 1967 Patodi joined the School of Mathematics of the Tata Institute of Fundamental Research in Bombay and he was to remain on the staff there until his death at the distressingly early age of 31 .

Mathematical fame for Patodi came early in his career with papers of great importance coming for the work of his Ph .D . His doctoral thesis, Heat equation and the index of elliptic operators, was supervised by M S Narasimhan and S Ramanan and the degree was awarded by the University of Bombay in 1971 .

Patodi s first paper Curvature and the eigenforms of the Laplace operator was part of his thesis and the contents of this paper are described in [2]: -

> *An analytic approach, via the heat equation yields easily a formula for the index of an elliptic operator on a compact manifold: but, the formula involves an integrand containing too many derivatives of the symbol, while from the Atiyah - Singer index theorem one would expect only two derivatives to figure .… Patodi s first contribution was to prove that such a fantastic cancellation of higher derivatives did indeed take place .*

The second paper which came from his thesis was An analytic proof of the Riemann - Roch - Hirzebruch theorem for

Kaehler manifolds which extended the methods of his first paper to a much more complicated situation.

The years 1971 to 1973 were ones which Patodi spent on leave at the Institute for Advanced Study at Princeton. There he worked with M F Atiyah and made several visits to work with others in his field at various centres in the United States and England. During this time he also collaborated with R Bott and I M Singer.

On his return to Bombay and the Tata Institute in 1973 Patodi was promoted to associate professor. He was promoted to full professor in 1976 but by this time his health was very poor. He had in fact had to overcome health problems for most of his career, making his achievements the more remarkable.

Patodi s publications, in addition to the two mentioned above, include a number of joint ones with Atiyah and Singer. These papers introduce a spectral invariant of a compact Riemannian manifold. In another paper he studies the relationship between Riemannian structures and triangulations. Other work gives a combinatorial formula for Pontryagin classes.

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K C Sreedharan Pillai

Born: 24 **Feb** 1920 **in Kerala, India**

Died: 5 **June** 1985 **in Lafayette, Indiana, USA**

Sree Pillai studied at the University of Travancore in Trivandrum. In 1937, just after Pillai began his studies there, the University of Travancore changed its name to the University of Kerala. He graduated in 1941 and obtained his Master s Degree in 1945.

Pillai was appointed a lecturer at the University of Kerala in 1945 and worked there for six years until he went to the United States in 1951. After studying for one year at Princeton, Pillai went to the University of North Carolina where he was awarded a doctorate in statistics in 1954.

His first post was as a statistician with the United Nations, a post he held from 1954 until 1962. Part of his duties in this post involved him founding the Statistical Center of the University of the Philippines. He was a visiting Professor and Advisor to the University over a number of years and supervised graduate students there.

In 1962 Pillai was appointed Professor of Statistics and Mathematics at Purdue University. In [2] his contributions to Purdue as described as follows: -

> *In the 23 years he served Purdue, he directed the research of 15 Ph.D. students. He was also an active consultant on several projects both within and outside the University. He was a close friend of his students and maintained a correspondence with most of them, some of whom are in remote parts of the world.*

Pillai s research was in statistics, in particular in multivariate statistical analysis. In [2] his work is described: -

> *… he obtained the probability distributions of statistics relating to several multivariate procedures.*

Perhaps his best known contribution is the widely used multivariate analysis of variance test which bears his name.

Pillai was honoured by being elected a Fellow of the American Statistical Association and a Fellow of the Institute of Mathematical Statistics. He was an elected member of the International Statistics Institute.

As well as his work at Purdue in developing the graduate programmes these Pillai was a keen golfer. This is described in [2]: -

His unique and unforgettable style charmed his playing companions and confused his opponents in the Purdue Staff League. His performances in the League matches were legendary.

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Prthudakasvami

Born: bout 830 in India

Died: bout 890 in India

Prthudakasvami is best known for his work on solving equations.

The solution of a first - degree indeterminate equation by a method called kuttaka (or "pulveriser") was given by Aryabhata I. This method of finding integer solutions resembles the continued fraction process and can also be seen as a use of the Euclidean algorithm.

Brahmagupta seems to have used a method involving continued fractions to find integer solutions of an indeterminate equation of the type $ax + c = by$. Prthudakasvami s commentary on Brahmagupta s work is helpful in showing how "algebra", that is the method of calculating with the unknown, was developing in India. Prthudakasvami discussed the kuttaka method which he renamed as "bijagnita" which means the method of calculating with unknown elements.

To see just how this new idea of algebra was developing in India, we look at the notation which was being used by Prthudakasvami in his commentary on Brahmagupta s Brahma Sputa Siddhanta. In this commentary Prthudakasvami writes the equation $10x + 8 = x^2 + 1$ as:

yava 0 ya 10 ru 8

yava 1 ya 0 ru 1

Here yava is an abbreviation for yavat avad varga which means the "square of the unknown quantity", ya is an abbreviation for yavat havat which means the "unknown quantity", and ru is an abbreviation for rupa which means "constant term". Hence the top row reads

$$0x^2 + 10x + 8$$

while the second row reads

$$x^2 + 0x + 1$$

The whole equation is therefore

$$0x^2 + 10x + 8 = x^2 + 0x + 1$$

or

$$10x + 8 = x^2 + 1.$$

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Cadambathur Tiruvenkatacharlu Rajagopal

Born: 8 **Sept** 1903 **in Triplicane, Madras, India**

Died: 25 **April** 1978 **in Madras, India**

Rajagopal was educated in Madras, India. He graduated in 1925 from the Madras Presidency College with Honours in mathematics.

He spent a short while in the clerical service, another short while teaching in Annamalai University then, from 1931 to 1951, he taught in the Madras Christian College. Here he gained an outstanding reputation as a teacher of classical analysis.

In 1951 Rajagopal was persuaded to join the Ramanujan Institute of Mathematics then, four years later, he became head of the Institute. Under his leadership the Institute became the major Indian mathematics research centre.

Rajagopal studied sequences, series, summability. He published 89 papers in this area generalising and unifying Tauberian theorems.

He also studied functions of a complex variable giving an analogue of a theorem of Edmund Landau on partial sums of Fourier series. In several papers he studied the relation between the growth of the mean values of an entire function and that of its Dirichlet series.

A final topic to interest him was the history of medieval Indian mathematics. He showed that the series for $\tan^{-1} x$ discovered by Gregory and those for sin x and cos x discovered by Newton were known to the Hindus 150 years earlier. He identified the Hindu mathematician Madhava as the first discoverer of these series.

Rajagopal is described in [1] as follows: -

> *Rajagopal was a teacher par excellence and a reliable and inspiring research guide. No words can*

adequately describe his modesty. Rational thinking and interest in psychic studies were two attributes which he imbibed with pride from his teacher Ananda Rau.

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Chidambaram Padmanabhan Ramanujam

Born: 9 **Jan** 1938 **in Madras, India**

Died: 27 **Oct** 1974 **in Bangalore, India**

Before we look at the life and work of Chidambaram Padmanabhan Ramanujam we must warn the reader that this article is on Ramanujam, NOT Ramanujan the number theorist who worked with G H Hardy (there is only a difference of one letter in their names!).

Ramanujam s father was C S Padmanabhan who was an advocate working in Madras, India, at the High Court. C P Ramanujam was educated in Madras, first at Ewart s School, where he had his primary and the first part of his secondary education, and then at the Sir M Ct Muthiah Chetty High School at Vepery, Madras. His interests on the academic side were in mathematics and chemistry while on the sporting side he was an enthusiastic tennis player. Chemistry experiments were particularly fascinating to him and he made a chemistry laboratory in a room in his home. There he would spend happy times carrying out experiments with one of his friends. In 1952, while still only 14 years old, he passed his final High School examinations and entered Loyola College in Madras.

Ramanujam s achievements at High School had been outstanding and he had shown that he was extraordinarily gifted, so he entered Loyola College with great expectations. He continued his interest in chemistry but it was mathematics that he specialised in, taking Mathematics Honours after obtaining his Intermediate qualification. He was awarded a B.A. with Honours in Mathematics in 1957 but, strangely for such an outstanding student, he only obtained a second class degree. This may have been a result of starting his university education at so young an age before he was really ready, for the second class degree no way reflected his remarkable mathematical abilities.

On the other hand it may have resulted from a lack of belief in himself which haunted Ramanujam throughout his life.

He had been taught mathematics by Father C Racine in his final honours years at Loyola College and he encouraged Ramanujam to apply for entry to the School of Mathematics at the Tata Institute in Bombay. In his letter of recommendation Father Racine wrote: -

> *He has certainly originality of mind and the type of curiosity which is likely to suggest that he will develop into a good research worker if given sufficient opportunity.*

In Madras there was another prestigious Institute, the Ramanujan Institute of Mathematics. In 1957 Ramanujam learnt deep results in analytic number theory from the former director of this Institute (who had retired three years earlier) in the months before he left Madras for Bombay to begin his studies at the Tata Institute. At the Institute, Ramanujam quickly became an expert in many different mathematical areas. His wide expertise made him a natural person to write up lecture notes from courses given by visitors to the Institute and in 1958 - 59 Max Deuring gave a course on the theory of algebraic functions of one variable which was expertly written up by Ramanujam.

He seemed able to soak up huge amounts of deep and difficult mathematics and he gave many talks showing what a deep understand he had of many topics. What he was not doing was producing original mathematical advances while some of his less able colleagues were being much more successful.

Ramanujam felt that he did not have what it takes to solve the big problems of mathematics, and he had no wish to solve small routine problems. Again, as in his undergraduate course, it would appear to be a psychological problem rather than a mathematical one but for Ramanujam it was a very real problem and he became more and more frustrated. He decided that his strengths were in teaching mathematics rather than producing original mathematics, and consequently he began applying to a variety of universities and colleges for a teaching position. His applications failed so reluctantly Ramanujam remained at the Tata Institute.

At this stage K G Ramanathan, the author of [4], began working with Ramanujam. He directed Ramanujam to work on some generalisations of the Waring problem to algebraic number fields. On this topic Ramanujam produced some outstanding results, generalising methods due to Davenport to attack certain questions which had been posed by Carl Siegel. For his deep results in number theory he was promoted to Associate Professor at the Tata Institute. It was not a position he easily accepted, arguing strongly that he was not worthy of such a post. However his friends and colleagues persuaded him to accept.

There is a fine line between whether someone behaves in a certain way because they have an illness or whether it is just their personality which determines their behaviour. Up to 1964 Ramanujam s lack of belief in his own abilities could have been described as part of his personality, but in 1964 he was struck with an illness which was diagnosed as severe depression and schizophrenia. Again feeling totally inadequate as a research mathematician he applied for university teaching posts.

During 1964 - 65 I R Shafarevich visited the Tata Institute and lectured on minimal models and birational transformations of two dimensional schemes. Ramanujam took notes at the lectures for publication and, as he had done previously he showed his deep understanding of mathematics in doing this task. On seeing the notes which Ramanujam had written, Shafarevich wrote to the Institute: -

> *I want to thank [Ramanujam] for the splendid job he has done. He not only corrected several mistakes but also complemented proofs of many results*

that were only stated in oral exposition. To mention some of them, he has written proofs of the Castelnuovo theorem…of the chain conditions…, the example of Nagata of a non-projective surface…and the proof of Zariski s theorem…

In July 1965 Ramanujam was offered a Professorship at the Punjab University in Chandigarh. He accepted and began teaching there. However his depression returned and [4]: -

> … *amidst tragic circumstances he had to cut short his stay there after about eight months.*

Back in the Tata Institute, Ramanujam received an invitation to spend six months at the Institut des Hautes tudes Scientifique in Paris. Again his illness forced him to return from Paris before the end of the six months. However his ability to do mathematics seemed as remarkable as ever outside his periods of illness. In 1967 - 68 David Mumford visited the Tata Institute and again Ramanujam wrote up his lectures for publication. In the Introduction to Abelian Varieties Mumford wrote: -

> … *these lectures were subsequently written up, and improved in many ways, by C P Ramanujam. The present text is a joint effort. … C P Ramanujam continuing my lectures at the Tata Institute lec-*

tured on and wrote up notes on Tate s theorem on homomorphisms between abelian varieties over finite fields.

Severe depression struck Ramanujam frequently. On one occasion he tried to take his life with barbiturates but was quickly treated and recovered. In February 1970, while again suffering depression, he resigned from the Tata Institute. The Director refused his resignation but later in the year he again resigned and went to the 1970 - 71 Algebraic geometry year at the University of Warwick in England. Mumford was also at the meeting and writes in [2]: -

> *… we were together in Warwick where he ran seminars on étale cohomology and on classification of surfaces. His excitement and enthusiasm was one of the main factors that made "Algebraic geometry year" a success. We discussed many topics involving topology and algebraic geometry at that time, and especially Kodaira s Vanishing Theorem. My wife and I spent many evenings together with him, talking about life, religion and customs both in India and the West and we looked forward to a warm and continuing friendship.*

As a result of his work with Shafarevich and Mumford, Ramanujam went on to make contributions to algebraic geometry which Mumford describes in [2]. These include a characterization of C^2, a version of the Kodaira vanishing theorem, a study of the automorphism group of a variety, a study of the purity of the discriminant locus, a proof that the invariance of the Milnor number implies the invariance of the topological type, and a geometric interpretation of multiplicity. The work on the Milnor number was done in collaboration with Le Dung Trang.

Back in India after his year at the University of Warwick, Ramanujam asked for a Professorship at the Tata Institute but be based in Bangalore where a new branch dealing with applications of mathematics was being set up. This was agreed and he taught analysis in Bangalore but, again in the depths of depression caused by his illness, he tried again to leave the Institute and obtain a university teaching post. While waiting for an offer of such a post from the Indian Institute at Simla he took his life with an overdose of barbiturates.

In [3] Ramanan pays this tribute to Ramanujam: -

For sheer elegance and economy, I have come across few mathematicians who were C P Ramanujam s equal. He made so many remarks which clarified *and threw light on different branches of mathematics that personally I derived immense pleasure from his company.*

Mumford writes in [2]: -

It was a stimulating experience to know and collaborate with C P Ramanujam. He loved mathematics and he was always ready to take up a new thread or pursue an old one with infectious enthusiasm. He was equally ready to discuss a problem with a first year student or a colleague, to work through an elementary point or puzzle over a deep problem. On the other hand he had high standards. He felt the spirit of mathematics demanded of him not merely routine developments but the right theorem an any given topic. He was sometimes tormented by these high standards, but, in retrospect, it is clear to us how often he succeeded in adding to our knowledge, results both new, beautiful and with a genuine original stamp.

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Srinivasa Aiyangar Ramanujan

Born: 22 Dec 1887 in Erode, Tamil Nadu state, India

Died: 26 April 1920 in Kumbakonam, Tamil Nadu state, India

Srinivasa Ramanujan was one of India s greatest mathematical geniuses. He made substantial contributions to the analytical theory of numbers and worked on elliptic functions, continued fractions, and infinite series.

Ramanujan was born in his grandmother s house in Erode, a small village about 400km southwest of Madras. When Ramanujan was a year old his mother took him to the town of Kumbakonam, about 160km nearer Madras. His father worked in Kumbakonam as a clerk in a cloth merchant s shop. In December 1889 he contracted smallpox.

When he was nearly five years old, Ramanujan entered the primary school in Kumbakonam although he would attend several different primary schools before entering the Town High School in Kumbakonam in January 1898. At the Town High School, Ramanujan was to do well in all his school subjects and showed himself an able all round scholar. In 1900 he began to work on his own on mathematics summing geometric and arithmetic series.

Ramanujan was shown how to solve cubic equations in 1902 and he went on to find his own method to solve the quartic. The following year, not knowing that the quintic could not be solved by radicals, he tried (and of course failed) to solve the quintic.

It was in the Town High School that Ramanujan came across a mathematics book by G S Carr called Synopsis of elementary results in pure mathematics. This book, with its very concise style, allowed Ramanujan to teach himself mathematics, but the style of the book was to have a rather unfortunate effect on the way Ramanujan was later to write down mathematics since it provided the only model that he had of written mathematical arguments. The book contained theorems, formulas and short proofs. It also contained an index to papers on pure mathematics which had been published in the European Journals of Learned Societies during the first half of the 19th century. The book, published in 1856, was of course well out of date by the time Ramanujan used it.

By 1904 Ramanujan had begun to undertake deep research. He investigated the series $\sum(1/n)$ and calculated Euler s constant to 15 decimal places. He began to study the Bernoulli numbers, although this was entirely his own independent discovery.

Ramanujan, on the strength of his good school work, was given a scholarship to the Government College in Kumbakonam which he entered in 1904. However the following year his scholarship was not renewed because Ramanujan devoted more and more of his time to mathematics and neglected his other subjects. Without money he was soon in difficulties and, without telling his parents, he ran away to the town of Vizagapatnam about 650km north of Madras. He continued his mathematical work, however, and at this time he worked on hypergeometric series and investigated relations between integrals and series. He was to discover later that he had been studying elliptic functions.

In 1906 Ramanujan went to Madras where he entered Pachaiyappa s College. His aim was to pass the First Arts examination which would allow him to be admitted to the University of Madras. He attended lectures at Pachaiyappa s College but became ill after three months study. He took the First Arts examination after having left the course. He passed in mathematics but failed all his other subjects and therefore failed the examination. This meant that he could not enter the University of Madras. In the following years he worked on mathematics developing his own ideas without any help and without any real idea of the then current research topics other than that provided by Carr s book.

Continuing his mathematical work Ramanujan studied continued fractions and divergent series in 1908. At this stage he became seriously ill again and underwent an operation in April 1909 after which he took him some considerable time to recover. He married on 14 July 1909 when his mother arranged for him to marry a ten - year - old girl S Janaki Ammal. Ramanujan did not live with his wife, however, until she was twelve years old.

Ramanujan continued to develop his mathematical ideas and began to pose problems and solve problems in the Journal of the Indian Mathematical Society. He developed relations between elliptic modular equations in 1910. After publication of a brilliant research paper on Bernoulli numbers in 1911 in the Journal of the Indian Mathematical Society he gained recognition for his work. Despite his lack of a university education, he was becoming well - known in the Madras area as a mathematical genius.

In 1911 Ramanujan approached the founder of the Indian Mathematical Society for advice on a job. After this he was appointed to his first job, a temporary post in the Accountant General s Office in Madras. It was then suggested that he approach Ramachandra Rao who was a Collector at Nellore. Ramachandra Rao was a founder member of the Indian Mathematical Society who had helped start the mathematics library. He writes in [30]: -

> *A short uncouth figure, stout, unshaven, not over clean, with one conspicuous feature - shining eyes - walked in with a frayed notebook under his arm. He was miserably poor. … He opened his book and began to explain some of his discoveries. I saw quite at once that there was something out of the*

way; but my knowledge did not permit me to judge whether he talked sense or nonsense. … I asked him what he wanted. He said he wanted a pittance to live on so that he might pursue his researches.

Ramachandra Rao told him to return to Madras and he tried, unsuccessfully, to arrange a scholarship for Ramanujan. In 1912 Ramanujan applied for the post of clerk in the accounts section of the Madras Port Trust. In his letter of application he wrote [3]: -

I have passed the Matriculation Examination and studied up to the First Arts but was prevented from pursuing my studies further owing to several untoward circumstances. I have, however, been devoting all my time to Mathematics and developing the subject.

Despite the fact that he had no university education, Ramanujan was clearly well - known to the university mathematicians in Madras for, with his letter of application, Ramanujan included a reference from E W Middlemast who was the Professor of Mathematics at The Presidency College in Madras. Middlemast, a graduate of St John s College, Cambridge, wrote [3]: -

I can strongly recommend the applicant. He is a young man of quite exceptional capacity in mathematics and especially in work relating to numbers. He has a natural aptitude for computation and is very quick at figure work.

On the strength of the recommendation Ramanujan was appointed to the post of clerk and began his duties on 1 March 1912. Ramanujan was quite lucky to have a number of people working round him with a training in mathematics. In fact the Chief Accountant for the Madras Port Trust, S N Aiyar, was trained as a mathematician and published a paper On the distribution of primes in 1913 on Ramanujan s work. The professor of civil engineering at the Madras Engineering College C L T Griffith was also interested in Ramanujan s abilities and, having been educated at University College London, knew the professor of mathematics there, namely M J M Hill. He wrote to Hill on 12 November 1912 sending some of Ramanujan s work and a copy of his 1911 paper on Bernoulli numbers.

Hill replied in a fairly encouraging way but showed that he had failed to understand Ramanujan s results on divergent series. The recommendation to Ramanujan that he read Bromwich s Theory of infinite series did not please Ramanujan much. Ramanujan wrote to E W Hobson and H F Baker trying to interest them in his results but neither replied. In January 1913 Ramanujan wrote to G H Hardy having seen a copy of his 1910 book Orders of infinity. In Ramanujan s letter to Hardy he introduced himself and his work [10]: -

> *I have had no university education but I have undergone the ordinary school course. After leaving school I have been employing the spare time at my disposal to work at mathematics. I have not trodden through the conventional regular course which is followed in a university course, but I am striking out a new path for myself. I have made a special investigation of divergent series in general and the results I get are termed by the local mathematicians as 'startling'.*

Hardy, together with Littlewood, studied the long list of unproved theorems which Ramanujan enclosed with his letter. On 8 February he replied to Ramanujan [3], the letter beginning: -

> *I was exceedingly interested by your letter and by the theorems which you state. You will however understand that, before I can judge properly of the*

value of what you have done, it is essential that I should see proofs of some of your assertions. Your results seem to me to fall into roughly three classes:

(1) *there are a number of results that are already known, or easily deducible from known theorems;*

(2) *there are results which, so far as I know, are new and interesting, but interesting rather from their curiosity and apparent difficulty than their importance;*

(3) *there are results which appear to be new and important…*

Ramanujan was delighted with Hardy s reply and when he wrote again he said [8]: -

I have found a friend in you who views my labours sympathetically. … I am already a half starving man. To preserve my brains I want food and this is my first consideration. Any sympathetic letter from you will be helpful to me here to get a scholarship either from the university of from the government.

Indeed the University of Madras did give Ramanujan a scholarship in May 1913 for two years and, in 1914, Hardy brought Ramanujan to Trinity College, Cambridge, to begin an extraordinary collaboration. Setting this up was not an easy matter. Ramanujan was an orthodox Brahmin and so was a strict vegetarian. His religion should have prevented him from travelling but this difficulty was overcome, partly by the work of E H Neville who was a colleague of Hardy s at Trinity College and who met with Ramanujan while lecturing in India.

Ramanujan sailed from India on 17 March 1914. It was a calm voyage except for three days on which Ramanujan was seasick. He arrived in London on 14 April 1914 and was met by Neville. After four days in London they went to Cambridge and Ramanujan spent a couple of weeks in Neville s home before moving into rooms in Trinity College on 30th April. Right from the beginning, however, he had problems with his diet. The outbreak of World War I made obtaining special items of food harder and it was not long before Ramanujan had health problems.

Right from the start Ramanujan s collaboration with Hardy led to important results. Hardy was, however, unsure how to approach the problem of Ramanujan s lack of formal education. He wrote [1]: -

> *What was to be done in the way of teaching him modern mathematics? The limitations of his knowledge were as startling as its profundity.*

Littlewood was asked to help teach Ramanujan rigorous mathematical methods. However he said ([31]): -

> *…that it was extremely difficult because every time some matter, which it was thought that Ramanujan needed to know, was mentioned, Ramanujan s response was an avalanche of original ideas which made it almost impossible for Littlewood to persist in his original intention.*

The war soon took Littlewood away on war duty but Hardy remained in Cambridge to work with Ramanujan. Even in his first winter in England, Ramanujan was ill and he wrote in March 1915 that he had been ill due to the winter weather and had not been able to publish anything for five months. What he did publish was the work he did in England, the decision having been made that the results he had obtained while in India, many of which he had communicated to Hardy in his letters, would not be published until the war had ended.

On 16 March 1916 Ramanujan graduated from Cambridge with a Bachelor of Science by Research (the degree was called a

Ph.D. from 1920). He had been allowed to enrol in June 1914 despite not having the proper qualifications. Ramanujan s dissertation was on Highly composite numbers and consisted of seven of his papers published in England.

Ramanujan fell seriously ill in 1917 and his doctors feared that he would die. He did improve a little by September but spent most of his time in various nursing homes. In February 1918 Hardy wrote (see [3]): -

> *Batty Shaw found out, what other doctors did not know, that he had undergone an operation about four years ago. His worst theory was that this had really been for the removal of a malignant growth, wrongly diagnosed. In view of the fact that Ramanujan is no worse than six months ago, he has now abandoned this theory - the other doctors never gave it any support. Tubercle has been the provisionally accepted theory, apart from this, since the original idea of gastric ulcer was given up. … Like all Indians he is fatalistic, and it is terribly hard to get him to take care of himself.*

On 18 February 1918 Ramanujan was elected a fellow of the Cambridge Philosophical Society and then three days later, the greatest honour that he would receive, his name appeared on the list for election as a fellow of the Royal Society of London. He had been proposed by an impressive list of mathematicians, namely Hardy, MacMahon, Grace, Larmor, Bromwich, Hobson, Baker, Littlewood, Nicholson, Young, Whittaker, Forsyth and Whitehead. His election as a fellow of the Royal Society was confirmed on 2 May 1918, then on 10 October 1918 he was elected a Fellow of Trinity College Cambridge, the fellowship to run for six years.

The honours which were bestowed on Ramanujan seemed to help his health improve a little and he renewed his effors at producing mathematics. By the end of November 1918 Ramanujan s health had greatly improved. Hardy wrote in a letter [3]: -

> *I think we may now hope that he has turned to corner, and is on the road to a real recovery. His temperature has ceased to be irregular, and he has gained nearly a stone in weight. … There has never been any sign of any dimination in his extraordinary mathematical talents. He has produced less, naturally, during his illness but the quality has been the same. …*
>
> *He will return to India with a scientific stand-*

ing and reputation such as no Indian has enjoyed before, and I am confident that India will regard him as the treasure he is. His natural simplicity and modesty has never been affected in the least by success - indeed all that is wanted is to get him to realise that he really is a success.

Ramanujan sailed to India on 27 February 1919 arriving on 13 March. However his health was very poor and, despite medical treatment, he died there the following year.

The letters Ramanujan wrote to Hardy in 1913 had contained many fascinating results. Ramanujan worked out the Riemann series, the elliptic integrals, hypergeometric series and functional equations of the zeta function. On the other hand he had only a vague idea of what constitutes a mathematical proof. Despite many brilliant results, some of his theorems on prime numbers were completely wrong.

Ramanujan independently discovered results of Gauss, Kummer and others on hypergeometric series. Ramanujan s own work on partial sums and products of hypergeometric series have led to major development in the topic. Perhaps his most famous work was on the number p(n) of partitions of an integer n into summands. MacMahon had produced tables of the value of p(n) for small numbers n, and Ramanujan used this numerical data to conjecture some remarkable properties some of which he proved using elliptic functions. Other were only proved after Ramanujan s death.

In a joint paper with Hardy, Ramanujan gave an asymptotic formula for p(n). It had the remarkable property that it appeared to give the correct value of p(n), and this was later proved by Rademacher.

Ramanujan left a number of unpublished notebooks filled with theorems that mathematicians have continued to study. G N Watson, Mason Professor of Pure Mathematics at Birmingham from 1918 to 1951 published 14 papers under the general title Theorems stated by Ramanujan and in all he published nearly 30 papers which were inspired by Ramanujan s work. Hardy passed on to Watson the large number of manuscripts of Ramanujan that he had, both written before 1914 and some written in Ramanujan s last year in India before his death.

The picture above is taken from a stamp issued by the Indian Post Office to celebrate the 75th anniversary of his birth.

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Sankara Narayana

Born: bout 840 in India

Died: bout 900 in India

Sankara Narayana (or Shankaranarayana) was an Indian astronomer and mathematician. He was a disciple of the astronomer and mathematician Govindasvami. His most famous work was the Laghubhaskariyavivarana which was a commentary on the Laghubhaskariya of Bhaskara I which in turn is based on the work of Aryabhata I.

The Laghubhaskariyavivarana was written by Sankara Narayana in 869 AD for the author writes in the text that it is written in the Shaka year 791 which translates to a date AD by adding 78. It is a text which covers the standard mathematical methods of Aryabhata I such as the solution of the indeterminate equation $by = ax \pm c$ (a, b, c integers) in integers which is then applied to astronomical problems. The standard Indian method involves using the Euclidean algorithm. It is called kuttakara (" pulveriser ") but the term eventually came to have a more general meaning like" algebra ". The paper [2] examines this method . The reader who is wondering what the determination of" mati "means in the title of the paper [2] then it refers to the optional number in a guessed solution and it is a feature which differs from the original method as presented by Bhaskara I .

Perhaps the most unusual feature of the Laghubhaskariyavivarana is the use of katapayadi numeration as well as the place - value Sanskrit numerals which Sankara Narayana frequently uses . Sankara Narayana is the first author known to use katapayadi numeration with this name but he did not invent it for it appears to be identical to a system invented earlier which was called varnasamjna . The numeration system varnasamjna was almost certainly invented by the astronomer Haridatta, and it was explained by him in a text which many historians believe was written in 684 but this would contradict what Sankara Narayana himself writes . This point is discussed below . First we should explain ideas behind Sankara Narayana s katapayadi numeration .

The system is based on writing numbers using the letters of the Indian alphabet . Let us quote from [1]: -

… the numerical attribution of syllables corresponds to the following rule, according to the regular order of succession of the letters of the Indian alphabet: the first nine letters represent the numbers 1 *to* 9 *while the tenth corresponds to zero; the following nine letters also receive the values* 1 *to* 9 *whilst the following letter has the value zero; the next five represent the first five units; and the last eight represent the numbers* 1 *to* 8 .

Under this system 1 to 5 are represented by four different letters . For example 1 is represented by the letters ka, ta, pa, ya which give the system its name (ka, ta, pa, ya becomes katapaya) . Then 6, 7, 8 are represented by three letters and finally nine and zero are represented by two letters .

The system was a spoken one in the sense that consonants and vowels which are not vocalised have no numerical value . The system is a place - value system with zero but one may reasonably ask why such an apparently complicated numeral system might ever come to be invented . Well the answer must be that it lead to easily remembered mnemonics . In fact many different" words "could represent the same number and this was highly useful for works written in verse as the Indian texts ten-ded to be.

Let us return to the interesting point about the date of Haridatta. Very unusually for an Indian text, Sankara Narayana expresses his thanks to those who have gone before him and developed the ideas about which he is writing. This in itself is not so unusual but the surprise here is that Sankara Narayana claims to give the list in chronological order. His list is

Aryabhata I

Varahamihira

Bhaskara I

Govindasvami

Haridatta

[Note that we have written Bhaskara I where Sankara Narayana simply wrote Bhaskara. The more famous Bhaskara II lived nearly 300 years after Sankara Narayana.]

Now the chronological order in the list agrees with the dates we have for the first four of these mathematicians. However, putting Haridatta after Govindasvami would seem an unlikely mistake for Sankara Narayana to make if Haridatta really did write his text in 684 since Sankara Narayana was himself a disciple of Govindasvami. If the dating given by Sankara Narayana is correct then katapayadi numeration had been invented only a few years before he wrote his text.

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Duncan MacLaren Young Sommerville

Born: 24 Nov 1879 in Beawar, Rajasthan, India
Died: 31 Jan 1934 in Wellington, New Zealand

Duncan Sommerville was the son of the Rev Dr James Sommerville. He was educated at Perth Academy (less than 50km from St Andrews) then at the University of St Andrews in Scotland. He was awarded a scholarship in 1899 to allow him to continue his studies at St Andrews. He taught there from 1902 to 1914 being appointed Lecturer in Mathematics in 1905.

Turnbull, writing in [3], describes Sommerville in these terms: -

> *His scholarly and unobtrusive demeanour as a young lecturer won the admiration of his colleagues and pupils in St Andrews where his teaching left a permanent mark. While he was essentially a geometer he had considerable interests in other sciences, and it is noteworthy that the classes which he chose to*

attend in his fourth year of study had been Anatomy and Chemistry. Crystallography in particular appealed to him, and doubtless these possible outlets influenced his geometrical concepts and led Sommerville to ponder over space filling figures, and gave an early impetus to thoughts in a field he made particularly his own. He had an original mind, and beneath his outward shyness considerable talents lay concealed: his intellectual grasp of geometry was balanced by a deftness in making models, and on the aesthetic side by an undoubted talent with the brush.

In 1915 Sommerville left Scotland for New Zealand to take up a new appointment as Professor of Pure and Applied Mathematics at Victoria College Wellington.

In 1919, when the professor of mathematics at Otago University suffered a nervous breakdown, a young student there A C Aitken was left without support and Sommerville began to tutor Aitken with a weekly correspondence.

Outside mathematics one of Sommerville s interests was astronomy and he was a founder of the New Zealand Astronomical Society as well as being its first secretary.

Sommerville worked on non - euclidean geometry and the history of mathematics. He proved in 1905 that there are eleven Archimedian tilings. His research was described by G Timmus as: -

> *… the classification of all types on non - euclidean geometry (including those usually excluded as bizarre), the extension, involving the measurement of generalised angles in higher space, of Euler s Theorem on polyhedra, space filling figures, the classification of polytopes (i.e. the generalisation, in higher space, of polyhedra), it is typical that this includes polytopes in non - euclidean space …*

In a review of [2] Daniel Coray states: -

> *By removing a finiteness condition which is habitually made on the angles formed by the various elements of a pencil (of lines, planes, etc.), Sommerville obtained more general geometries than the usual ones (Euclid, Lobachevsky, Riemann). He classified them into 9 types of plane geometries, 27 in dimension 3, and more generally 3^n in dimension n. A number of these geometries have found applications, for instance in physics.*

In 1911 he published Bibliography of non - Euclidean Geometry, including the Theory of Parallels, the Foundations of Geometry and Space of n Dimensions. There are 1832 references to n - dimensional geometry .

Books which Sommerville published were Elements of Non - Euclidean Geometry (1914) , Analytic Conics (1924), Introduction to Geometry of n dimensions (1929) and Three Dimensional Geometry (1934) . He also wrote 30 papers on combinatorial geometry .

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Sridhara

Born: bout 870 in possibly Bengal, India

Died: bout 930 in India

Sridhara is now believed to have lived in the ninth and tenth centuries. However, there has been much dispute over his date and in different works the dates of the life of Sridhara have been placed from the seventh century to the eleventh century. The best present estimate is that he wrote around 900 AD, a date which is deduced from seeing which other pieces of mathematics he was familiar with and also seeing which later mathematicians were familiar with his work. We do know that Sridhara was a Hindu but little else is known. Two theories exist concerning his birthplace which are far apart. Some historians give Bengal as the place of his birth while other historians believe that Sridhara was born in southern India.

Sridhara is known as the author of two mathematical treatises, namely the Trisatika (sometimes called the Patiganitasara) and the Patiganita . However at least three other works have been attributed to him, namely the Bijaganita, Navasati, and Brhatpati . Information about these books was given the works of Bhaskara II (writing around 1150), Makkibhatta (writing in 1377), and Raghavabhatta (writing in 1493) . We give details below of Sridhara s rule for solving quadratic equations as given by Bhaskara II .

There is another mathematical treatise Ganitapancavimsi which some historians believe was written by Sridhara . Hayashi in [7], however, argues that Sridhara is unlikely to have been the author of this work in its present form .

The Patiganita is written in verse form . The book begins by giving tables of monetary and metrological units . Following this algorithms are given for carrying out the elementary arithmetical operations, squaring, cubing, and square and cube root extraction, carried out with natural numbers . Through the whole book Sridhara gives methods to solve problems in terse rules in verse form which was the typical style of Indian texts at this time . All the algorithms to carry out arithmetical operations are presented in this way and no proofs are given . Indeed there is no suggestion that Sridhara realised that proofs are in any way necessary . Often after stating a rule Sridhara gives one or more numerical examples, but he does not give solutions to these example nor does he even give answers in this work.

After giving the rules for computing with natural numbers, Sridhara gives rules for operating with rational fractions. He gives a wide variety of applications including problems involving ratios, barter, simple interest, mixtures, purchase and sale, rates of travel, wages, and filling of cisterns. Some of the examples are decidedly non-trivial and one has to consider this as a really advanced work. Other topics covered by the author include the rule for calculating the number of combinations of n things taken m at a time. There are sections of the book devoted to arithmetic and geometric progressions, including progressions with a fractional numbers of terms, and formulas for the sum of certain finite series are given.

The book ends by giving rules, some of which are only approximate, for the areas of a some plane polygons. In fact the text breaks off at this point but it certainly was not the end of the book which is missing in the only copy of the work which has survived. We do know something of the missing part, however, for the Patiganitasara is a summary of the Patiganita including the missing portion.

In [7] Shukla examines Sridhara s method for finding rational solutions of $Nx^2 \pm 1 = y^2$, $1 - Nx^2 = y^2$, $Nx^2 \pm C = y^2$, and $C - Nx^2 = y^2$ which Sridhara gives in the Patiganita. Shukla states that the rules given there are different from those given by other Hindu mathematicians.

Sridhara was one of the first mathematicians to give a rule to solve a quadratic equation. Unfortunately, as we indicated above, the original is lost and we have to rely on a quotation of Sridhara s rule from Bhaskara II: -

> *Multiply both sides of the equation by a known quantity equal to four times the coefficient of the square of the unknown; add to both sides a known quantity equal to the square of the coefficient of the unknown; then take the square root.*

To see what this means take

$ax^2 + bx = c$.

Multiply both sides by 4a to get

$4a^2x^2 + 4abx = 4ac$

then add b^2 to both sides to get

$4a^2x^2 + 4abx + b^2 = 4ac + b^2$

and, taking the square root

$2ax + b = \sqrt{4ac + b^2}$.

There is no suggestion that Sridhara took two values when he took the square root .

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Sripati

Born: 1019 in (probably) Rohinikhanda, Maharashtra, India

Died: 1066 in India

Sripati s father was Nagadeva (sometimes written as Namadeva) and Nagadeva s father, Sripati s paternal grandfather, was Kesava. Sripati was a follower of the teaching of Lalla writing on astrology, astronomy and mathematics. His mathematical work was undertaken with applications to astronomy in mind, for example a study of spheres. His work on astronomy was undertaken to provide a basis for his astrology. Sripati was the most prominent Indian mathematicians of the 11th Century.

Among Sripati s works are: Dhikotidakarana written in 1039, a work of twenty verses on solar and lunar eclipses; Dhruvamanasa written in 1056, a work of 105 verses on calculating planetary longitudes, eclipses and planetary transits; Siddhantasekhara a major work on astronomy in 19 chapters; and Ganitatilaka an incomplete arithmetical treatise in 125 verses based on a work by Sridhara .

The titles of Chapters 13, 14, and 15 of the Siddhanta-sekhara are Arithmetic, Algebra and On the Sphere . Chapter 13 consists of 55 verses on arithmetic, mensuration, and shadow reckoning . It is probable that the lost portion of the arithmetic treatise Ganitatilaka consisted essentially of verses 19 - 55 of this chapter . The 37 verses of Chapter 14 on algebra state various rules of algebra without proof . These are given in verbal form without algebraic symbols . In verses 3, 4 and 5 of this chapter Sripati gave the rules of signs for addition, subtraction, multiplication, division, square, square root, cube and cube root of positive and negative quantities . His work on equations in this chapter contains the rule for solving a quadratic equation and, more impressively, he gives the identity:

$$x + y = \frac{x + x^2 - y}{2} + \frac{x - x^2 - y}{2}$$

Other mathematics included in Sripati s work includes, in particular, rules for the solution of simultaneous indeterminate equations of the first degree that are similar to those given by Brahmagupta

Sripati obtained more fame in astrology than in other areas and it is fair to say that he considered this to be his most important contributions. He wrote the Jyotisaratnamala which was an astrology text in twenty chapters based on the Jyotisaratnakosa of Lalla. Sripati wrote a commentary on this work in Marathi and it is one of the oldest works to have survived that is written in that language. Marathi is the oldest of the regional languages in Indo-Aryan, dating from about 1000.

Another work on astrology written by Sripati is the Jatakapaddhati or Sripatipaddhati which is in eight chapters and is [1]: -

> … *one of the fundamental textbooks for later Indian genethlialogy, contributing an impressive elaboration to the computation of the strengths of the planets and astrological places. It was enormously popular, as the large number of manuscripts, commentaries, and imitations attests.*

Now genethlialogy was the science of casting nativities and it was the earliest branch of astrology which claimed to be able to predict the course of a person s life based on the positions of the planets and of the signs of the zodiac at the moment the person was born or conceived.

There is one other work on astrology the Daivajnavallabha which some historians claim was written by Sripati while other claim that it is the work of Varahamihira. As yet nobody has come up with a definite case to show which of these two is the author, or even whether the author is another astrologer.

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Varahamihira

Born: 505 **in Kapitthaka, India**

Died: 587 **in India**

Our knowledge of Varahamihira is very limited indeed. According to one of his works, he was educated in Kapitthaka. However, far from settling the question this only gives rise to discussions of possible interpretations of where this place was. Dhavale in [3] discusses this problem. We do not know whether he was born in Kapitthaka, wherever that may be, although we have given this as the most likely guess. We do know, however, that he worked at Ujjain which had been an important centre for mathematics since around 400 AD. The school of mathematics at Ujjain was increased in importance due to Varahamihira working there and it continued for a long period to be one of the two leading mathematical centres in India, in particular having Brahmagupta as its next major figure.

The most famous work by Varahamihira is the Pancasiddhantika (The Five Astronomical Canons) dated 575 AD. This work is important in itself and also in giving us information about older Indian texts which are now lost. The work is a treatise on mathematical astronomy and it summarises five earlier astronomical treatises, namely the Surya, Romaka, Paulisa, Vasistha and Paitamaha siddhantas. Shukla states in [11]: -

> *The Pancasiddhantika of Varahamihira is one of the most important sources for the history of Hindu astronomy before the time of Aryabhata II.*

One treatise which Varahamihira summarises was the Romaka - Siddhanta which itself was based on the epicycle theory of the motions of the Sun and the Moon given by the Greeks in the 1st century AD. The Romaka - Siddhanta was based on the tropical year of Hipparchus and on the Metonic cycle of 19 years. Other works which Varahamihira summarises are also based on the Greek epicycle theory of the motions of the heavenly bodies. He revised the calendar by updating these earlier works to take into account precession since they were written. The Pancasiddhantika also contains many examples of the use of a place - value number system.

There is, however, quite a debate about interpreting data from Varahamihira s astronomical texts and from other similar works. Some believe that the astronomical theories are Babylonian in origin, while others argue that the Indians refined the Babylonian models by making observations of their own. Much needs to be done in this area to clarify some of these interesting theories.

In [1] Ifrah notes that Varahamihira was one of the most famous astrologers in Indian history. His work Brihatsamhita (The Great Compilation) discusses topics such as [1]:-

> *… descriptions of heavenly bodies, their movements and conjunctions, meteorological phenomena, indications of the omens these movements, conjunctions and phenomena represent, what action to take and operations to accomplish, sign to look for in humans, animals, precious stones, etc.*

Varahamihira made some important mathematical discoveries. Among these are certain trigonometric formulas which translated into our present day notation correspond to

$\sin x = \cos(\pi/2 - x)$,

$\sin^2 x + \cos^2 x = 1$, and

$(1 - \cos 2x)/2 = \sin^2 x$.

Another important contribution to trigonometry was his sine tables where he improved those of Aryabhata I giving more accurate values. It should be emphasised that accuracy was very important for these Indian mathematicians since they were computing sine tables for applications to astronomy and astrology. This motivated much of the improved accuracy they achieved by developing new interpolation methods.

The Jaina school of mathematics investigated rules for computing the number of ways in which r objects can be selected from n objects over the course of many hundreds of years. They gave rules to compute the binomial coefficients ${}_nC_r$ which amount to

$$ {}_nC_r = n(n-1)(n-2)\cdots(n-r+1)/r! $$

However, Varahamihira attacked the problem of computing ${}_nC_r$ in a rather different way. He wrote the numbers n in a column with n = 1 at the bottom. He then put the numbers r in rows with r = 1 at the left-hand side. Starting at the bottom left side of the array which corresponds to the values n = 1, r = 1, the values of ${}_nC_r$ are found by summing two entries, namely the one directly below the (n, r) position and the one immediately to the left of it. Of course this table is none other than Pascal s triangle for finding the binomial coefficients despite being viewed from a different angle from the way we build it up today. Full details of this work by Varahamihira is given in [5].

Hayashi, in [6], examines Varahamihira s work on magic squares. In particular he examines a pandiagonal magic square of order four which occurs in Varahamihira s work.

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Vijayanandi

Born: bout 940 in Benares (now Varanasi), India

Died: bout 1010 in India

Vijayanandi (or Vijayanandin) was the son of Jayananda. He was born into the Brahman caste which meant he was from the highest ranking caste of Hindu priests. He was an Indian mathematician and astronomer whose most famous work was the Karanatilaka. We should note that there was another astronomer named Vijayanandi who was mentioned by Varahamihira in one of his works. Since Varahamihira wrote around 550 and the Karanatilaka was written around 966, there must be two astronomers both named "Vijayanandi".

The Karanatilaka has not survived in its original form but we know of the text through an Arabic translation by al-Biruni. It is a work in fourteen chapters covering the standard topics of Indian astronomy. It deals with the topics of: units of time measurement; mean and true longitudes of the sun and moon; the length of daylight; mean longitudes of the five planets; true longitudes of the five planets; the three problems of diurnal rotation; lunar eclipses, solar eclipses; the projection of eclipses; first visibility of the planets; conjunctions of the planets with each other and with fixed stars; the moon s crescent; and the patas of the moon and sun.

The Indians had a cosmology which was based on long periods of time with astronomical events occurring a certain whole number of times within the cycles. This system led to much work on integer solutions of equations and their application to astronomy. In particular there was, according to Aryabhata I, a basic period of 4320000 years called a mahayuga and it was assumed that the sun, the moon, their apsis and node, and the planets reached perfect conjunctions after this period. Hence it was assumed that the periods were rational multiples of each other.

All the planets and the node and apsis of the moon and sun had to have an integer number of revolutions in the mahayuga. Many Indian astronomers proposed different values for these integral numbers of revolutions. For the number of revolutions of the apsis and node of the moon per mahayuga, Aryabhata I proposed 488219 and 232226, respectively.

However Vijayanandi changed these numbers to 488211 and 232234. The reasons for giving the new numbers is unclear. Like other Indian astronomers, Vijayanandi made contributions to trigonometry and it appears that his calculation of the periods was computed by using tables of sines and versed sines. It is significant that accuracy was need in trigonometric tables to give accurate astronomical theories and this motivated many of the Indian mathematicians to produce more accurate methods of approximating entries in tables.

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John Henry Constantine Whitehead

Born: 11 **Nov** 1904 **in Madras, India**

Died: 8 **May** 1960 **in Princeton, New Jersey, USA**

Henry Whitehead s father was The Right Rev Henry Whitehead, Bishop of Madras in India .His mother, Isobel Duncan, was the daughter of the Rector of Calne, Wiltshire, so Henry came from a family deeply involved with the Church . However, Henry Whitehead s family also had strong academic connections; in particular there was a strong tradition of mathematical excellence .His mother had studied mathematics at Oxford University, being one of the early women undergraduates, while the famous mathematician and philosopher, Alfred North Whitehead, was his uncle.

Although Henry was born in India he lived in England from the age of about eighteen months. It was at that age that his parents brought him back from India and left him in the care of his maternal grandmother who lived in Oxford. His parents then returned to India and Henry saw little of them while he was growing up. It would not be until his father retired when Henry was sixteen years old that they returned to England.

Henry s childhood in Oxford was a quiet one since [6]: -

> *… it was a very peaceful place and he would recall going for drives with his grandmother in her carriage and seeing the horse - drawn buses in town.*

Henry did quite well at primary school, both academically, socially and in sport. He was [6]: -

> *… of above average intelligence, good at games, prone to be careless in his work, but with a great capacity for enjoying life. If he had worked harder he might have won a scholarship to Eton…*

Despite the lack of a scholarship to Eton, Whitehead was successful in the Entrance Examination and began a happy period at Eton where he specialised in mathematics, yet never showed himself as a mathematical genius. One reason why this outstanding mathematician only appeared "good" at school was a whole range of other interests which occupied him [6]: -

> *His exuberance, gaiety and intelligence made him many friends and his irrepressible high spirits and disregard for authority sometimes strained the patience of his tolerant and long suffering housemaster. His personal popularity got him elected to Pop, and his athletic prowess won him a place in the cricket second eleven, his fives colours and a silver cup at boxing.*

Another reason why he failed to shine academically may have been due to an inner sadness at being separated from his parents. Whatever the reasons it made his desired progress harder than it might otherwise have been. Whitehead wanted to go to Balliol College, Oxford, to study mathematics but his mathematics teacher at Eton did not think that he stood a chance of winning a scholarship. He wrote [6]: -

> *In pure geometry he has not been over diligent … he would have been more successful in mathematics if he had been less so at cricket.*

The mathematics teacher was wrong, however, and in

March 1923 Whitehead did win a scholarship to study at Balliol College. At Balliol Whitehead was tutored by J W Nicholson, who had been a student of Whitehead s uncle A N Whitehead. However, Nicholson s health was poor and Whitehead was tutored frequently by H Newboult at Merton College.

As Eton had done, however, Oxford also provided a whole range of distractions to Whitehead. He played many sports including cricket, squash, tennis, and boxing. His interest in cricket brought him into contact with G H Hardy, so the sporting interests had some academic benefits. At Oxford Whitehead developed another passion, namely playing poker. He claimed that his mother taught him how to play the game when he was a young child recovering from an illness. Whitehead played poker for quite large sums of money while at Oxford although his friends did not always pay what they owed him.

At Oxford Whitehead showed himself to be better than the "good" at mathematics which he had displayed at Eton. Despite his success, and the award of a First Class degree, he did not consider himself sufficiently talented for an academic career so, in 1927, he joined the firm of stockbrokers Buckmaster and Moore. By this time his parents had returned from India and were living in the village of Sulham in Berkshire and Whitehead lived there and travelled to his job in the city of London every day.

It took not much over a year of work at the stockbrokers to convince Whitehead that the City was not the life for him so, in 1928, he returned to the University of Oxford. While at Oxford Whitehead met Veblen, who was on leave from Princeton. He attended a seminar which Veblen gave on differential geometry and it must have been a very fine talk for it persuaded Whitehead that he would undertake research in that topic. Veblen supported Whitehead s application for a Commonwealth Fellowship to enabled him to study for a Ph.D. at Princeton.

Whitehead arrived at Princeton in the summer of 1929 to begin his research. He worked mainly on differential geometry although towards the end of his three years there he became interested in topology. He was awarded his doctorate from Princeton in 1932 for a dissertation entitled The Representation of Projective Spaces. Whitehead s joint work with his doctoral supervisor Veblen led to The Foundations of Differential Geometry (1932), now considered a classic. It contains the first proper definition of a differentiable manifold.

As we mentioned Whitehead s interests turned more towards topology near the end of his three years in Princeton when he collaborated with Lefschetz in proving that all analytic manifolds can be triangulated. In this area he is best remembered for his work on homotopy equivalence. The three years at Princeton were happy ones for Whitehead and he [6]: -

> *… had throughout his life a really deep affection for [Princeton] and its inhabitants, ranging from the Dean of the Graduate College to the barman at "Andy s".*

Whitehead returned to Oxford after being awarded his doctorate and he was elected to a Fellowship at Balliol College in 1933. In the following year Whitehead married Barbara Sheila Carew Smyth (one of the authors of [6]) who was a concert pianist [3]: -

> *They shared great zest for life and enjoyed a marriage of surpassing happiness.*

They lived at first in St Giles, Oxford, but later moved to North Oxford after their first of their two sons was born. Their home there [6]: -

> *… became a meeting place for mathematicians, where there was generally a mug of beer or a cup of tea and always a warm welcome, and a pencil and a block of paper each for host and guest to write their*

thoughts on. Many ideas were exchanged and many informal seminars took place in his study.

Soon after his return to England, Whitehead wrote another major work on differential geometry On the Covering of a Complete Space by the Geodesics Through a Point (1935). Whitehead also studied Stiefel manifolds and set up a school of topology at Oxford. However, events would soon cause a break in Whitehead s career.

The Nazi moves against Jewish mathematicians gave Whitehead great distress, and he actively helped many to escape to safety. In particular he helped Eilenberg and Dehn, while Schr dinger came to live in his home after escaping from Austria. Whitehead left Oxford in 1940 to undertake war work in London, spending [6]: -

> …*the night of the worse blitz on London sitting in his friend s wine cellar placidly working at mathematics. He afterwards congratulated himself on his high standard of morality as not one bottle was* [*opened*].

After working at the Board of Trade, at the Admiralty, and finally at the Foreign Office, during the War, Whitehead returned to his home in North Oxford when World War II ended . In 1947 he was appointed to the Waynflete Chair of Pure Mathematics at Oxford . At that time Whitehead moved from Balliol College to Magdalen College .

Whitehead s father died in 1947 and his mother died six years later in 1953 . She had owned a small farm and when Whitehead inherited the cattle he and his wife decided to buy Manor Farm in Noke, north of Oxford . The farm was run mainly by Whitehead s wife but he took a keen interest in the farm where the couple lived until Whitehead s death . His death, while on a visit to the Institute for Advanced Study at Princeton during a Sabbatical year he was spending in the United States, was totally unexpected [6]: -

> *In May* 1960, *without any previous warning symptoms or illness, he died of a heart attack in Princeton, where his mathematical life had begun .*

Whitehead s personality is clearly described in [6]: -

> *He was able to reach across the barriers of age, class and nationality to talk on equal terms with anyone who shared his passion for mathematics . The long series of collaborative papers written between* 1950 *and* 1960 *reflects his eagerness to share his ideas and to interest himself in the results of others,*

which remained undiminished to the end of his life. It was in long mathematical conversations, in which ever detail had to be hammered out till he had it quite correct and secure that he most delighted and it is by these conversations, gay and informal, in which he contrived to make everyone his own equal, that he will be best remembered by those who knew him.

His career produced mathematical results which would have a major influence on the directions of mathematical research. However, as the authors of [6] relate, his influence extended beyond this: -

His influence on the development of mathematics during his active lifetime can be partly measured by the innumerable references, implicit and explicit, in current mathematical literature on algebraic and geometric topology; but it could not have been so great without the generosity and enthusiasm which he poured into every mathematical enterprise and which inspired such deep affection in all who knew him well.

Whitehead received several honours for outstanding mathematical achievements, but he died at the age of 55 when at the height of his powers, so did not live to receive awards which normally come later in life. He was elected to the Royal Society in 1944. He served the London Mathematical Society in a number of ways, most notably as president during 1953 - 55.

Max Newman, who was a friend of Henry Whitehead from 1929 until the end of his life, writes [3]: -

> *The immediate attractiveness of his tremendous high spirits and friendly manners would not have sufficed to bring him the lasting affection of mathematical friends all over the world if it had not been backed, from his earliest days, by a most delicate perception of the thoughts and feelings of the person he was talking to, and a deep enjoyment and tolerance of all kinds of human behaviour. He had the dislike of formality which is not uncommon among men of science and learning, but it was a comfortable, not an uncomfortable, informality which enabled him to soften the high and exalted as easily as he could unfreeze the young and timid. Those leisurely, searching conversations, enjoyed on exactly the same terms by all comers, on a walk over the*

farm, with his not very obedient gun - dog, or sitting in arm chairs with pencils and blocks of paper for following the details, were as refreshing and enlivening after 30 years as on the first day .

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Yativrsabha

Born: bout 500 in India

Died: bout 570 in India

Yativrsabha (or Jadivasaha) was a Jaina mathematician who studied under Arya Manksu and Nagahastin. We know nothing of Yativrsabha s dates except for a reference which he makes to the end of the Gupta dynasty which he says was after 231 years of ruling. This ended in 551 so we must assume that 551 AD is a date which occured during Yativrsabha s lifetime. This fits with the only other information regarding his dates which are that his work is referenced by Jinabhadra Ksamasramana in 609 and that Yativrsabha himself refers to a work written by Sarvanandin in 458.

Yativrsabha s work Tiloyapannatti gives various units for measuring distances and time and also describes the system of infinite time measures. It is a work which describes Jaina cosmology and gives a description of the universe which is of historical importance in understanding Jaina science and mathematics. The Jaina belief was in an infinite world, both infinite in space and in time. This led the Jainas to devise ways of measuring larger and larger distances and longer and longer intervals of time. It led them to consider different measures of infinity, and in this respect the Jaina mathematicians would appear to be the only ones before the time when Cantor developed the theory of infinite cardinals to envisage different magnitudes of infinity.

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Yavanesvara

Born: bout 120 in Western India

Died: bout 180 in India

Indian astrology was originally known as Jyotisha, which means“ science of the stars ”. Until around the first century AD no real distinction was made between astrology and astronomy and in fact most astronomical theories were propounded to support the theory that the positions of the heavenly bodies directly influenced human events .

The Indian methods of computing horoscopes all date back to the translation of a Greek astrology text into Sanskrit prose by Yavanesvara in 149 AD . Yavanesvara (or Yavanaraja) literally means“ Lord of the Greeks ”and it was a name given to many officials in western India during the period 130 AD - 390 AD . During this period the Ksatrapas ruled Gujarat (or Madhya Pradesh) and these“ Lord of the Greeks ”officials acted for the Greek merchants living in the area .

The particular "Lord of the Greeks" official Yavanesvara who we are interested in here worked under Rudradaman. Rudradaman became ruler of the Ksatrapas in around 130 AD and it was during the period of his rule that Yavanesvara worked as an official and made his translation. We know of Rudradaman because information is recorded in a lengthy San-skrit inscription at Junagadh written around 150 AD.

The Greek astrology text in question was written in Alexandria some time round about 120 BC. Yavanesvara did far more than just translate the Greek text for such a translation would have had little relevance to the Indians. He therefore not only translated the language but he translated the context too. Instead of the Greek gods who appear in the original, Yavanesvara used Hindu images. Again he worked the Indian caste system into the work and made the work one which would fit well with the Indian thought.

The work was written with the aim of letting Indians became astrologers so it had to present astronomy in a form in which it could be used for astrology. In order to do this Yavanesvara put into his work an explanation of the Greek version of the Babylonian theory of the motions of the planets. All this he wrote in Sanskrit prose but sadly the original has not survived. We do have, however, a version written in Sanskrit verse 120 years after Yavanesvara s work appeared.

Yavanesvara had an important influence on the whole of astrology in India for centuries after he made his popular translation. Although the influence was more than on astrology, as the science of astronomy split from astrology, the influence of Yavanesvara s work reached into astronomy too.

References for Yavanesvara

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