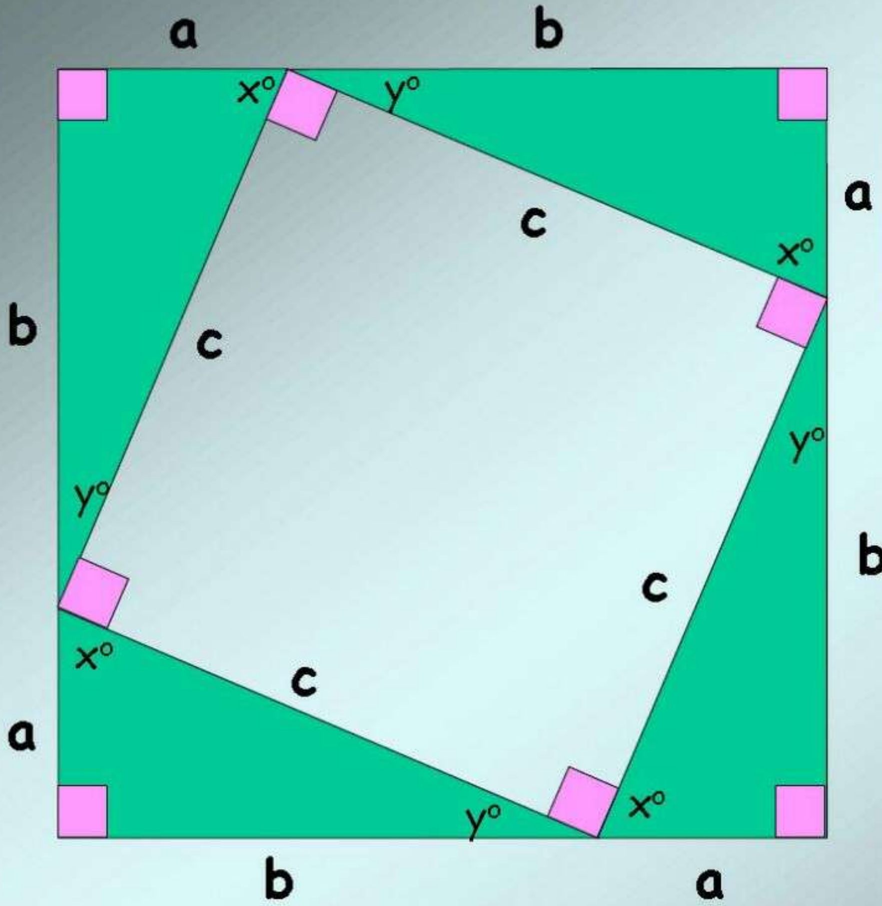
A Proof of Pythagoras Theorem

To prove that $a^2 + b^2 = c^2$



We first need to show that the shape in the middle is a square.

- The sides are equal in length since each is the hypotenuse of congruent triangles. ✓
- The angles are all 90° since x+y = 90° and angles on a straight line add to 180° ✓

Area of large square

$= (a + b)^2 = a^2 + 2ab + b^2$

Area of large square is also

$= c^2 + 4 \times \frac{1}{2} ab = c^2 + 2ab$

So

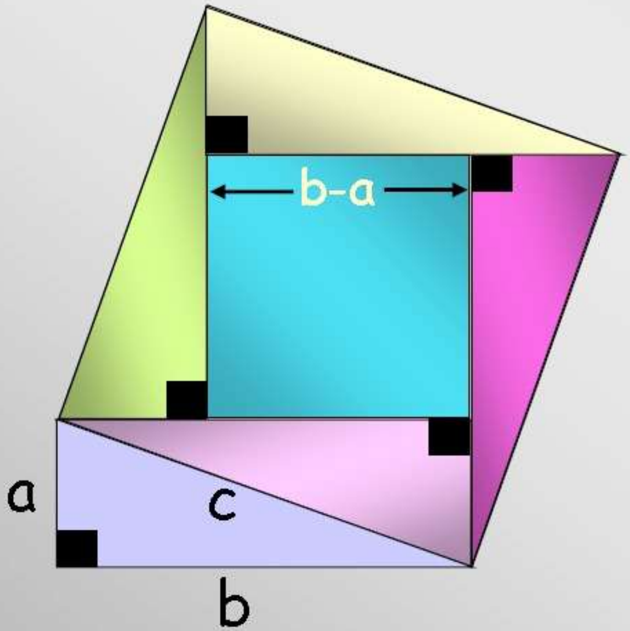
$\Rightarrow a^2 + 2ab + b^2 = c^2 + 2ab$

$\Rightarrow a^2 + b^2 = c^2$ QED

Take 3 identical copies of this right-angled triangle and arrange like so.

Bhaskara's Proof (Indian Mathematician 12th century)

Bhaskara's approach is to partition the square on the hypotenuse into 4 right-angled triangles that are congruent to the original, plus a central square.



To prove that $a^2 + b^2 = c^2$

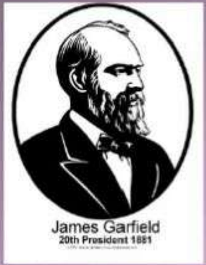
$c^2 = 4 \times \frac{1}{2} ab + (b-a)^2$

$c^2 = 2ab + b^2 - 2ab + a^2$

$c^2 = a^2 + b^2$ (QED)

President James Garfield's Proof(1876)

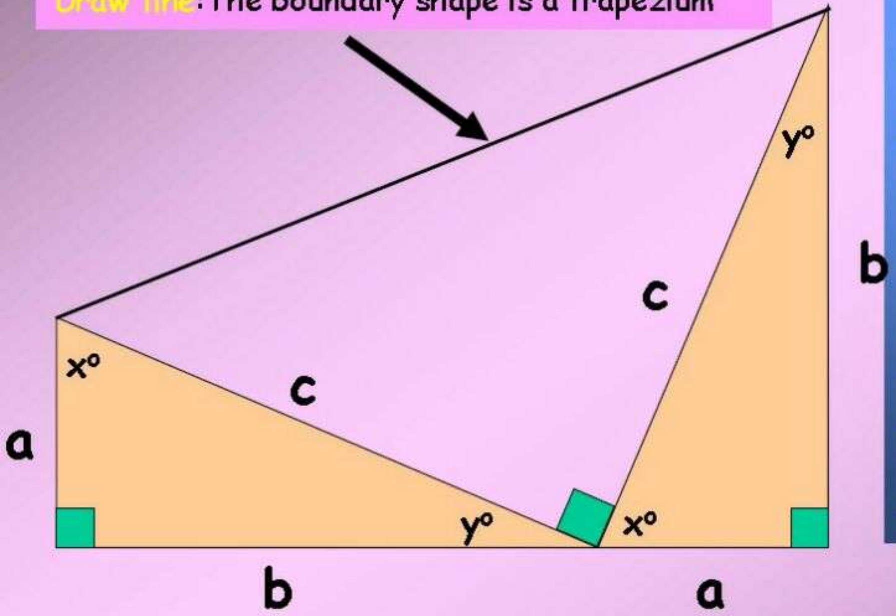
To prove that $a^2 + b^2 = c^2$



We first need to show that the angle between angle x and angle y is a right angle.

- This angle is 90° since $x + y = 90°$ and angles on a straight line add to 180° ✓

Draw line: The boundary shape is a trapezium



Area of trapezium

$= \frac{1}{2}(a + b)(a + b) = \frac{1}{2}(a^2 + 2ab + b^2)$

Area of trapezium is also equal to the areas of the 3 right-angled triangles.

$= \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$

So

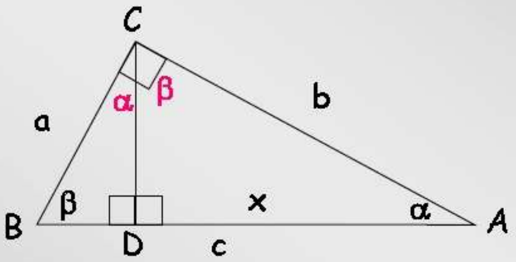
$\Rightarrow \frac{1}{2}(a^2 + 2ab + b^2) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$

$\Rightarrow a^2 + 2ab + b^2 = 2ab + c^2$

$\Rightarrow a^2 + b^2 = c^2$ QED

Take 1 identical copy of this right-angled triangle and arrange like so

John Wallis Proof: English Mathematician (1616-1703)



Draw CD perpendicular to AB

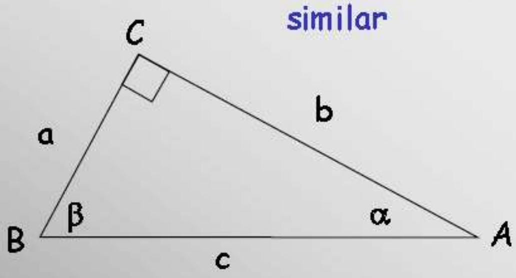
Angle BDC is a right angle (angles on a straight line)

Angle BCD = α since $\alpha + \beta + 90° = 180°$ (from large triangle)

Angle ACD= β = since $\alpha + \beta + 90° = 180°$ (from large triangle)

All 3 triangles are similar since they are equiangular

Triangles ACB, CDB and ADC are similar



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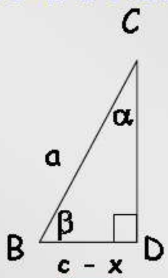

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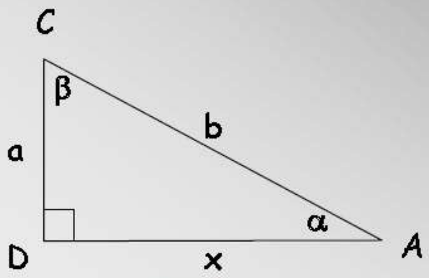

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Comparing corresponding sides in 1 and 2:

$$\frac{a}{c-x} = \frac{c}{a} \Rightarrow a^2 = c^2 - cx$$

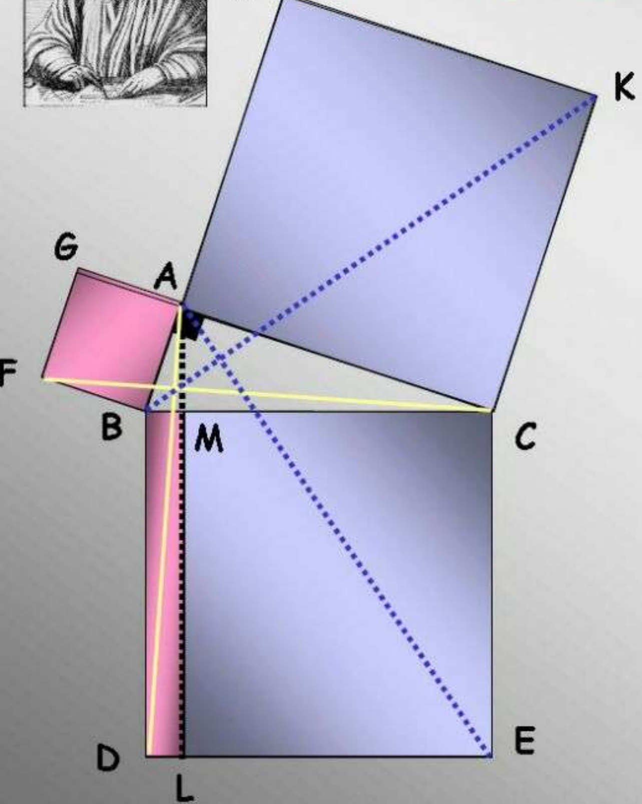
Comparing corresponding sides in 1 and 3:

$$\frac{b}{x} = \frac{c}{b} \Rightarrow b^2 = cx$$

adding equations gives: $a^2 + b^2 = c^2$

Euclid's Proof

To Prove that area of square BDEC = area of square ABFG + area of square ACHK



Proof:

- Construct squares on each of the 3 sides (1.46)
- Draw AL through A parallel to BD (1.31)
- Draw Lines AD and FC

- CA and AG lay on the same straight line (2 right angles)(1.14)
- In triangles ABD and FBC AB = FB (sides of the same small square)
- BD = BC (sides of the same larger square)
- Also included angles are equal (right angle + common angle ABC)
- ∴ triangles are congruent (SAS) and so are equal in area (1.4)
- Rectangle BDLM = 2 × area of triangle ABD (1.41)
- Square ABFG = 2 × area of triangle FBC (1.41)
- **∴ Area of rectangle BDLM = Area of square ABFG**

Draw lines BK and AE

- BA and AH lay on the same straight line (2 right angles (1.14)
- In triangles ACE and BCK, AC = CK (sides of smaller square)
- BC = CE (sides of larger square)
- Also included angles are equal (right angle + common angle ACB)
- ∴ triangles are congruent (SAS) and so are equal in area (1.4)

Rectangle MLCE = 2 × area of triangle Ace (1.41)

Square ACHK = 2 area of triangle BCK (1.41)

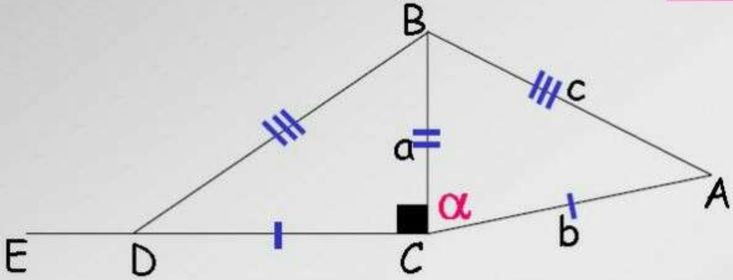
- **∴ Area of rectangle MLCE = Area of square ACHK**

Area of square BDEC = area of square ABFG + area of square ACHK. QED

Euclid's Proof of the Converse of Pythagoras' Theorem (I.48)

To prove that: If the square on the hypotenuse is equal to the sum of the squares on the other two sides then the triangle **contains a right angle**.

The Proof



To prove that angle α is a right angle

Given $c^2 = a^2 + b^2$

- Draw CE perpendicular to BC
- Construct CD **equal** to CA and join B to D

Applying Pythagoras' Theorem to triangle BCD

$BD^2 = BC^2 + DC^2$ (I.47)

$\Rightarrow BD^2 = a^2 + b^2$ (since $BC = a$ and $DC = b$)

$\Rightarrow BD^2 = c^2$ (since $a^2 + b^2 = c^2$ **given**)

$\Rightarrow BD = c$

$\Rightarrow$ Triangles BCD and BCA are congruent by (SSS) $\therefore$ angle α is a right angle QED